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Elementary Spinor Theory

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The basic part of the theory of spinors is developed by elementary means with the use of a simple picture of a spinor and spherical trigonometry. The topics treated are the geometrical description of a spinor, components of a spinor, the equations for the rotation of a spinor, determination of the magnitude function of a spinor, and addition and multiplication of spinors. The relation of spinors to quaternions is briefly discussed. No detailed treatment of the physical applications of spinor theory is given.

1.

THE theory of spinors,[1] known chiefly for its application to electron spin and relativistic quantum mechanics, was worked out by B. L. van der Waerden in 1929. The basic principles of spinor theory have, however, been known for many years; indeed, some of its formulas (Euler's rotational parameters[2]) date back to 1776. The main forerunner of present-day spinor theory was Hamilton's quaternion theory[3] (about 1845); we shall see later how the two are related to each other.

Spinors have other applications besides those already mentioned. The components of a unit spinor have been used in the mechanics of rigid bodies for more than fifty years under the name of "Cayley-Klein parameters."[2] Spinors appear to be closely related to the theory of the electromagnetic field, although efforts to construct a complete electromagnetic theory solely on the basis of spinors[4] have not, as yet, been generally regarded as successful. Lastly, spinor theory is connected with the theory of electric transmission lines.[5]

Unfortunately, the available literature on spinors is exceedingly hard to understand. Most of it presupposes a knowledge of modern algebra, especially the theory of groups, and all of it presents the subject in such an abstract and indirect way that the reader, even if he succeeds in following the reasoning, is not likely to get any intuitive picture of what is going on. The object

[1] For information on spinors and related matters see van der Waerden, *Die gruppentheoretische Methode in der Quantenmechanik* (Julius Springer, Berlin, Germany, 1932), pp. 57–87; Weyl, *Gruppentheorie und Quantenmechanik* (S. Hirzel, Leipzig, Germany, 1931), second edition, pp. 121–133; Kramers, *Quantentheorie des Elektrons und der Strahlung* (Akademische Verlagsgesellschaft, 1938), pp. 258–280; Wigner, *Gruppentheorie und ihre Anwendung* (Friedrich Vieweg & Sohn, Braunschweig, Germany, 1931), pp. 164–183; Cartan, *Leçons sur la Theorie des spineurs* (Hermann & Cie, Paris, France, 1938); also, O. Laporte and G. E. Uhlenbeck, Phys. Rev. **37**, 1380 (1931).

[2] Whittaker, *Analytical Dynamics* (Dover Publications, New York, 1944), fourth edition, pp. 8–12.

[3] For information on quaternions see the article by Murnaghan in the book *A Collection of Papers in Memory of Sir William Rowan Hamilton* (Scripta Mathematica, New York, 1945). Also, F. Klein, *Entwicklung der Mathematik* (Chelsea Publishing Company, New York, 1950), Vol. 1, pp. 184–191.

[4] L. de Broglie, *Une nouvelle théorie de la lumière* (Hermann & Cie, Paris, France, 1940–42).

[5] L. Brillouin, J. phys. et radium **7**, 401 (1936).

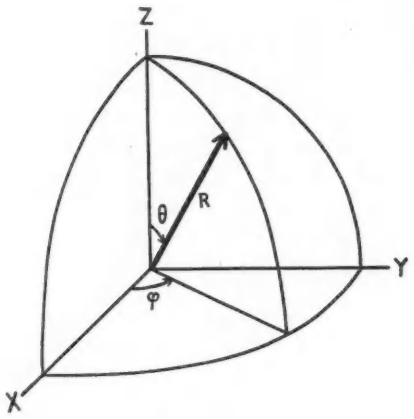

FIG. 1. Vector.

of the present paper is to try to give a direct and elementary treatment of the basic part of spinor theory. The development parallels to some extent the treatment of vectors in standard works on vector analysis, and instead of using group theory, is based on a simple picture of a spinor and on spherical trigonometry.

2. GEOMETRICAL DESCRIPTION OF A SPINOR

In discussing vectors and tensors there are two ways in which we can proceed: the geometrical and the analytical. To use the geometrical approach, we describe each kind of quantity in terms of its magnitudes and directions; in the analytical treatment we use components. Thus, we describe a vector geometrically by saying that it has magnitude and direction (with sense), and we use an arrow to represent it pictorially. Similarly, we may describe a symmetric tensor geometrically by saying that it has three magnitudes and three mutually perpendicular directions (without sense); we can use three perpendicular double-headed arrows as a picture of this. We shall now describe a spinor by saying that it has magnitude, direction (with sense), and angular position about the axis of the direction. To get a picture of a spinor we can think of an ax, letting the handle represent the magnitude, direction, and sense just as the arrow does in the case of the vector; once we have fixed these specifications, the ax still has one degree of freedom left, since it can still be rotated about the axis of the handle; this last degree of freedom is what we have called angular position about the axis. To draw a picture of a spinor we shall draw a conventionalized ax consisting of a line for the handle and a short line segment, perpendicular to the handle, for the blade. The magnitude of the spinor is measured by the length of the handle, but no meaning is to be attached to the length of the blade, just as, in the picture of a vector no meaning is attached to the size of the arrowhead. Figure 1 shows a vector of magnitude R with Cartesian coordinate axes, and the polar angle θ and the azimuthal angle φ of the vector are also shown. Figure 2 is the same kind of diagram for a spinor of magnitude S and angles θ, φ, and ψ, the angle ψ indicating the angular position of the spinor about its axis. Note that ψ is the angle between the *southbound* local meridian and the spinor blade, measured from the meridian to the blade. It will be considered positive if its sense is counterclockwise, as it is in Fig. 2. The angles θ, φ, and ψ specify the position of the spinor in the same way in which the three Eulerian angles specify the position of a rigid body having a fixed point; so we shall call θ, φ, and ψ the Eulerian angles of the spinor.[6]

3. COMPONENTS OF A SPINOR

The analytical description of vectors and tensors is based on the use of components, which are functions of the magnitudes and directions

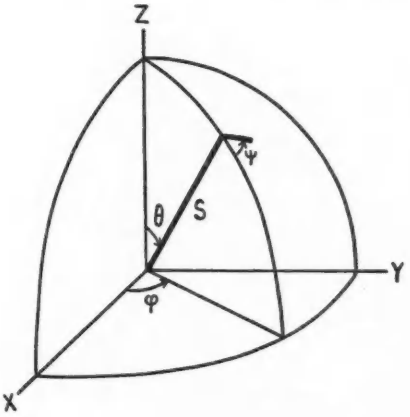

FIG. 2. Spinor.

[6] Our angle φ is equal to the usual second Eulerian angle plus 90°, and our ψ is equal to the usual third Eulerian angle minus 90°. Note that some authors interchange the meanings of φ and ψ.

involved and also of the coordinate axes used. Perhaps the most important property of the components is that they transform by simple equations if we rotate the coordinate axes. In order to treat spinors analytically, we must, analogously, find functions of the specifications of a spinor (i.e., of S, θ, φ, and ψ) that will transform simply if we rotate the coordinate axes. It will be more convenient for making diagrams, however, if we leave the coordinate axes fixed and rotate the spinor instead; this will not alter the problem, since the values of the components will depend only on the *relative* positions of the spinor and the coordinate axes anyway. Our problem, then, is this: given a spinor with initial specifications S, θ, φ, and ψ, and with final specifications S', θ', φ', and ψ' after it has been rotated in any arbitrary way, we wish to find functions of S', θ', φ', and ψ' that can be expressed by means of simple formulas in terms of the same functions of S, θ, φ, and ψ. Presumably there should be four such functions, since a spinor has one more degree of freedom than a vector, but we shall see that the four functions can be gotten together into only two functions if complex quantities are used.

It would seem from the foregoing that we must study all possible rotations of a spinor; however, every possible rotation can be duplicated by a suitable combination of rotations about the z axis and about the y axis (as will be shown later); and therefore, we need study only these two special kinds of rotations.

Let us first rotate the spinor about the z axis, say through the angle φ_r. (We shall use the subscript r to designate a *rotation* angle.) Evidently S, θ, and ψ will be unchanged and φ will be increased by φ_r. This information is of no help in the problem of finding functions qualified to serve as components of the spinor.

Let us next rotate the spinor about the y axis through an angle θ_r. Evidently S will be unchanged. The rotation is shown in Fig. 3; the initial and final positions of the spinor blade are at B and D, respectively, and the spinor handle is not shown. We have taken the spinor blade to lie along the great circle that intersects the y axis; the loss of generality entailed by this arrangement is trivial. The initial angles of the spinor are

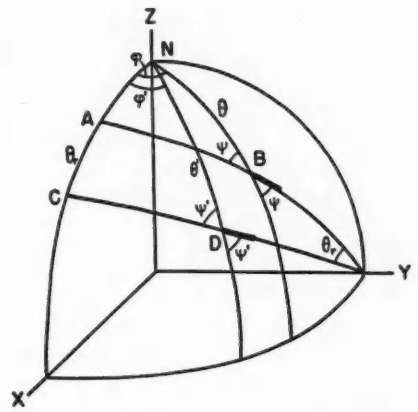

FIG. 3. Rotation of spinor about y-axis.

θ, φ, and ψ, the final angles θ', φ', and ψ', and the angles θ, θ', and θ_r are denoted by the arcs NB, ND, and AC. We wish to find relations between functions of θ, φ, and ψ and functions of θ', φ', and ψ'. This is a simple problem in spherical trigonometry; we merely apply the four relations known as Delambre's analogies[7] to each of the spherical triangles NAB and NCD (taking φ, ψ, θ, or φ', ψ', θ', for the two angles and the included side) and make use of the facts that the arcs AB and CD are equal, the angles NAB and NCD are right angles, and the arc NC is equal to the arc AN plus θ_r. A short computation yields the following results:

$$\left.\begin{aligned}
&\cos\tfrac{1}{2}\theta' \cos\tfrac{1}{2}(\varphi'+\psi') \\
&\quad=\cos\tfrac{1}{2}\theta_r \cos\tfrac{1}{2}\theta \cos\tfrac{1}{2}(\varphi+\psi) \\
&\qquad-\sin\tfrac{1}{2}\theta_r \sin\tfrac{1}{2}\theta \cos\tfrac{1}{2}(\psi-\varphi) \\
&\cos\tfrac{1}{2}\theta' \sin\tfrac{1}{2}(\varphi'+\psi') \\
&\quad=\cos\tfrac{1}{2}\theta_r \cos\tfrac{1}{2}\theta \sin\tfrac{1}{2}(\varphi+\psi) \\
&\qquad-\sin\tfrac{1}{2}\theta_r \sin\tfrac{1}{2}\theta \sin\tfrac{1}{2}(\psi-\varphi) \\
&\sin\tfrac{1}{2}\theta' \cos\tfrac{1}{2}(\psi'-\varphi') \\
&\quad=\sin\tfrac{1}{2}\theta_r \cos\tfrac{1}{2}\theta \cos\tfrac{1}{2}(\varphi+\psi) \\
&\qquad+\cos\tfrac{1}{2}\theta_r \sin\tfrac{1}{2}\theta \cos\tfrac{1}{2}(\psi-\varphi) \\
&\sin\tfrac{1}{2}\theta' \sin\tfrac{1}{2}(\psi'-\varphi') \\
&\quad=\sin\tfrac{1}{2}\theta_r \cos\tfrac{1}{2}\theta \sin\tfrac{1}{2}(\varphi+\psi) \\
&\qquad+\cos\tfrac{1}{2}\theta_r \sin\tfrac{1}{2}\theta \sin\tfrac{1}{2}(\psi-\varphi)
\end{aligned}\right\}. \quad (1)$$

If the second and fourth of these equations are multiplied by the imaginary unit i and added to the first and third, respectively, the results will

[7] See any standard textbook of spherical trigonometry.

be expressed in only two equations

$$\left.\begin{aligned}\cos\tfrac{1}{2}\theta' e^{i(\varphi'+\psi')/2} &= \cos\tfrac{1}{2}\theta_r \cos\tfrac{1}{2}\theta e^{i(\varphi+\psi)/2} \\ &\quad -\sin\tfrac{1}{2}\theta_r \sin\tfrac{1}{2}\theta e^{i(\psi-\varphi)/2} \\ \sin\tfrac{1}{2}\theta' e^{i(\psi'-\varphi')/2} &= \sin\tfrac{1}{2}\theta_r \cos\tfrac{1}{2}\theta e^{i(\varphi+\psi)/2} \\ &\quad +\cos\tfrac{1}{2}\theta_r \sin\tfrac{1}{2}\theta e^{i(\psi-\varphi)/2}\end{aligned}\right\}. \quad (2)$$

From these equations we can see that the functions

$$\cos\tfrac{1}{2}\theta e^{i(\varphi+\psi)/2} \quad \text{and} \quad \sin\tfrac{1}{2}\theta e^{i(\psi-\varphi)/2} \quad (3)$$

undergo a homogeneous linear transformation with coefficients

$$\begin{pmatrix}\cos\tfrac{1}{2}\theta_r & -\sin\tfrac{1}{2}\theta_r \\ \sin\tfrac{1}{2}\theta_r & \cos\tfrac{1}{2}\theta_r\end{pmatrix}, \quad (4)$$

when the spinor is rotated through θ_r about the y axis and that they are transformed by it into the *same* functions of θ', φ', and ψ'. The functions (3), however, are not quite suitable to use as components of the spinor, because they do not depend on the spinor's magnitude S. To get the components of the spinor, therefore, we shall multiply these two functions by a function of the magnitude $f(S)$ whose form will be determined later. Thus, if we call the components S_1 and S_2, then we shall have

$$\begin{aligned}S_1 &= f(S)\cos\tfrac{1}{2}\theta e^{i(\varphi+\psi)/2} \\ S_2 &= f(S)\sin\tfrac{1}{2}\theta e^{i(\psi-\varphi)/2}.\end{aligned} \quad (5)$$

We shall call $f(S)$ the *magnitude function*. (The *magnitude* is just S.) The transformation Eqs. (2) can now be written more simply

$$\begin{aligned}S_1' &= S_1\cos\tfrac{1}{2}\theta_r - S_2\sin\tfrac{1}{2}\theta_r \\ S_2' &= S_1\sin\tfrac{1}{2}\theta_r + S_2\cos\tfrac{1}{2}\theta_r.\end{aligned}$$

We must now reconsider the rotation of the spinor about the z axis to see whether it can be described in terms of the components defined by Eq. (5). Evidently, it can be so described by means of a linear transformation with coefficients

$$\begin{pmatrix}e^{i\varphi_r/2} & 0 \\ 0 & e^{-i\varphi_r/2}\end{pmatrix}; \quad (6)$$

for this transformation, when applied to S_1 and S_2, will add φ_r to φ and leave S, θ, and ψ unchanged, just as it should.

We may note here that the functions (3) are sometimes called the *Cayley-Klein parameters* associated with the Eulerian angles θ, φ, and ψ.[8]

Figure 4 shows five unit spinors (i.e., spinors whose magnitude function is unity) in different positions, as computed on the basis of Eqs. (5). Note that the spinors with components (1,0) and (0,1) are *antiparallel* to each other. These spinors are called the *basis spinors*. Their counterparts in vector theory (i.e., the unit basis vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$) are *perpendicular* to one another. In spinor theory it is a straight angle, rather than a right angle, that plays the role of orthogonality. We shall call the two basis spinors $\mathbf{u}$ and $\mathbf{v}$, as shown in Fig. 4. Any arbitrary spinor can be expressed as a linear combination of $\mathbf{u}$ and $\mathbf{v}$

$$\mathbf{S} = S_1\mathbf{u} + S_2\mathbf{v}; \quad (7)$$

just as any arbitrary vector can be expressed as a linear combination of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, the coefficients being the components of the vector.

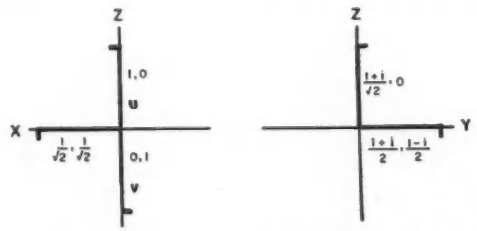

FIG. 4. Some spinors and their components.

4. GENERAL ROTATION OF A SPINOR

Thus far we have found the analytical expressions for rotations of a spinor about the z axis and about the y axis only. To find the equations that express an arbitrarily chosen rotation, let us think of the rotation as acting on *all* spinors, and note that its effect on any given spinor will be completely determined (except for sense) by the specifications of the end position into which it carries the *one* spinor $\mathbf{u}$. Let this end position have angles θ_r, φ_r, and ψ_r; then these will be called the Eulerian angles of the rotation. Now it is easy to see that a rotation with Eulerian angles θ_r, φ_r, and ψ_r can be duplicated by the following sequence: first, a rotation about the z axis through the angle ψ_r, next, a rotation about the

[8] See Whittaker or Kramers (reference 2 or 1). Our functions (3) differ slightly from the usual Cayley-Klein parameters, because we have used slightly different conventions; see reference 6.

y axis through θ_r, and finally, a rotation about the z axis through φ_r. To see this, note that the first rotation sets $\mathbf{u}$ at an angle ψ_r with the meridian in the xz plane; and the second and third bring $\mathbf{u}$ to the latitude and longitude, respectively, of its final position.

It follows from the foregoing that if *any* spinor is subjected to a rotation with Eulerian angles θ_r, φ_r, and ψ_r, its components will be transformed in exactly the same way as if they had been subjected first to a transformation of the type (6) (but with the angle ψ_r instead of φ_r in the exponents), next to a transformation of the type (4), and lastly to another transformation of the type (6), with the angle φ_r. According to the theory of linear transformations, the resultant of these three transformations will have for its matrix (i.e., table of coefficients) the product of the matrices of the three individual transformations. This matrix product, evaluated from the matrices (4) and (6) according to the rule for multiplying matrices (with the matrix involving ψ_r on the *right*) comes out to be

$$\begin{pmatrix} \cos\frac{1}{2}\theta_r e^{i(\varphi_r+\psi_r)/2} & -\sin\frac{1}{2}\theta_r e^{i(\varphi_r-\psi_r)/2} \\ \sin\frac{1}{2}\theta_r e^{i(\psi_r-\varphi_r)/2} & \cos\frac{1}{2}\theta_r e^{-i(\varphi_r+\psi_r)/2} \end{pmatrix}. \quad (8)$$

The quantities in the first column of this matrix[9] are evidently the components of the final position into which the rotation has carried the basis spinor $\mathbf{u}$, so we shall call these quantities u_1' and u_2'. The quantities in the second column are the components of the final position of the other basis spinor, and we shall call them v_1' and v_2'. Thus the matrix of the arbitrary rotation is

$$\begin{pmatrix} u_1' & v_1' \\ u_2' & v_2' \end{pmatrix}. \quad (9)$$

The fact that the four coefficients of a rotation are the components of the two basis spinors in their final positions is the exact analog of the fact that in ordinary solid analytic geometry the nine coefficients of an orthogonal transformation are the components of the three new basis vectors referred to as the three old basis vectors.

[9] The real and imaginary parts of these two quantities are Euler's rotational parameters, already mentioned in the first paragraph of this paper.

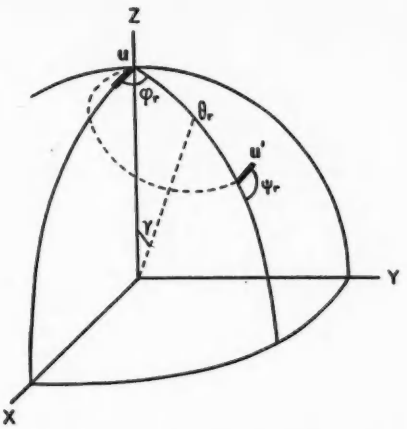

FIG. 5. Rotation of spinor through 180°.

5. ROTATION IN TERMS OF AXIS AND ANGLE

Instead of describing a rotation in terms of its effect on the basis spinors, we can describe it in terms of the direction of the axis of rotation and the angle of rotation. Let us first consider the special case of a 180° rotation about any arbitrary axis having α, β, and γ for its direction angles. This will carry $\mathbf{u}$ to a new position $\mathbf{u}'$. Figure 5 shows the blades of $\mathbf{u}$ and $\mathbf{u}'$; the dotted line is the rotation axis, and the dotted semicircle is the path along which the blade of $\mathbf{u}$ moves. From the figure we see that $\varphi_r+\psi_r=180°$, θ_r is equal to twice the polar angle of the rotation axis (or to 2γ), and φ_r is equal to the azimuthal angle of the rotation axis. If we apply these statements to the general rotation matrix (8), we will get matrix (8) expressed in terms of the polar and azimuthal angles of the rotation axis; then matrix (8) can easily be expressed in terms of α, β, and γ by standard formulas. The resulting matrix for the 180° rotation is

$$\begin{pmatrix} i\cos\gamma & i\cos\alpha-\cos\beta \\ i\cos\alpha+\cos\beta & -i\cos\gamma \end{pmatrix},$$

which can be written thus,

$$\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\cos\alpha+\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\cos\beta + \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\cos\gamma. \quad (10)$$

An immediate corollary to this result is the following theorem, important in spin theory: If **A** is any vector, then the linear transformation whose matrix is

$$\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} A_x + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} A_y + \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} A_z \quad (11)$$

rotates all spinors 180° about the axis of **A** and multiplies their magnitude functions by the magnitude of **A**. It follows that the eigenspinors of (11) (i.e., the spinors that on having the transformation (11) applied to them are merely multiplied by a constant instead of being transformed in a more complicated way) are the spinors that are parallel or antiparallel to **A**, as one can see from Eqs. (5), noting that such a spinor gets only its ψ—not its θ or φ—changed by the rotation. This mathematical construction, an operator whose eigenspinors express parallelism and antiparallelism, respectively, to a given vector, is exactly what the experimental facts of electron spin phenomena call for. Actually, the matrix (11) is not quite suitable to use as the operator of spin theory because its eigenvalues are imaginary; however, we can get an operator with real eigenvalues by simply multiplying (11) through by i.

The three matrices in (10) are the matrices of 180° rotations about the x, y, and z axes, as is seen by setting one of the angles α, β, or γ equal to zero. The similarity in form between (10) and the expression for an arbitrary unit *vector* with direction angles α, β, and γ, *viz.*,

$$\mathbf{i}\cos\alpha + \mathbf{j}\cos\beta + \mathbf{k}\cos\gamma,$$

is striking. However, (10) is not a vector in the modern sense of the word, for its square is not a positive number. In fact, the square of (10) is

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (12)$$

as is readily verified by multiplying (10) by itself term by term, observing the rules of matrix multiplication. Now the square of a 180° rotation is geometrically a 360° rotation, and so we conclude that every 360° rotation, irrespective of the axis direction, multiplies the components of a spinor by -1. A second 360° rotation would restore the original values. A spinor, then, must complete two laps before it has run through its entire cycle of values; its components are double-valued functions of position. Our ax picture affords no way of telling which lap a spinor is on, but for most purposes this causes no particular difficulty. An advantage in the two-lap description of rotations is that it discriminates between two rotations of opposite senses, even when they are 180° rotations; the ordinary description of rotations by orthogonal transformations of the Cartesian coordinates cannot do this.

We shall next find the matrix of a rotation through any arbitrary angle, say ω_r. Suppose first that the rotation is about the z axis; then its matrix will be (6) (but with ω_r instead of φ_r) and this can be written

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cos\tfrac{1}{2}\omega_r + \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \sin\tfrac{1}{2}\omega_r. \quad (13)$$

In words, the matrix is a linear combination of the identity matrix and the matrix of a 180° rotation about the z axis, the coefficients being the cosine and the sine of half the rotation angle. Since this statement is independent of the direction of the z axis, it must be true for any arbitrary axis, and therefore, the matrix of a rotation through ω_r about an axis with direction angles α, β, and γ is, according to (10),[10]

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cos\tfrac{1}{2}\omega_r + \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \sin\tfrac{1}{2}\omega_r \cos\alpha + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \sin\tfrac{1}{2}\omega_r \cos\beta + \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \sin\tfrac{1}{2}\omega_r \cos\gamma. \quad (14)$$

6. RELATION TO QUATERNION THEORY

A transformation that a spinor undergoes on being rotated, i.e., a transformation having either of the forms (8) or (14) is called a *versor*. A *quaternion* is the product of a versor by any

[10] The coefficients in the expression (14) are the Eulerian parameters of the rotation (see reference 9) now expressed in terms of the rotation angle and the direction cosines of the rotation axis. Anyone who dislikes the reasoning by which we have derived expression (14) should note that it is possible to obtain (14) by a direct computation similar to that which led to (10) but more complicated.

positive real number, known as the *tensor*[11] of the quaternion. Thus a quaternion represents geometrically a combined rotation of spinors and multiplication of their magnitudes, the multiplication factor being unity in the case of a versor. Quaternion theory, then, is the study of a certain group of transformations, while spinor theory is primarily concerned with the quantities to which these transformations can be applied.

Hamilton's original formulation of quaternion theory was much less explicit than our discussion would suggest. Matrices were not yet in common use,[12] and so instead of the four matrices appearing in our formula (14) Hamilton used four abstract operators denoted by the symbols $1, i, j, k$,[11] which he defined by means of the system of postulates (which we would nowadays call a group multiplication table,[13] though it is not a complete one),

$$\begin{aligned} &i^2=j^2=k^2=-1; \quad ij=-ji=k; \\ &jk=-kj=i; \qquad\quad ki=-ik=j. \end{aligned} \tag{15}$$

These equations, more than a hundred years old, are nearly identical with the basic equations in Pauli's formulation of the theory of electron spin and might well be regarded as the prototype of many of the commutation rules appearing in present-day quantum mechanics.

It is easily verified that the four matrices in (14) satisfy Eqs. (15). However, there are many other combinations of matrices that will satisfy Eqs. (15) equally well; we could have gotten any one of these in our formula (14) by taking for our three basic rotations, not 180° rotations about the x, y, and z axes, but rather 180° rotations about three other mutually perpendicular axes. In the language of group theory the matrices in (14) form only one of the many possible representations of Eqs. (15).

The extreme abstractness of Hamilton's formulation made quaternion theory unintelligible to many, and it is perhaps the emulation of this abstractness on the part of modern writers on spinor theory that makes the existing literature on spinors so difficult to read.

7. DETERMINATION OF THE MAGNITUDE FUNCTION

The magnitude function $f(S)$, Eqs. (5), can evidently not be determined by a study of rotations. So far as we can see at this stage, it can be any arbitrary function of S, but it should be so chosen, of course, that the resulting spinor theory will be of maximum usefulness in physics. Since most of the laws of physics involve vectors, this means that $f(S)$ should be so chosen that the components of spinors can be simply expressed in terms of the components of vectors. The best way to do this, as we shall see, is to take $f(S)$ to be $(2S)^{\frac{1}{2}}$. Then the components of a spinor are

$$\begin{aligned} S_1&=(2S)^{\frac{1}{2}}\cos\tfrac{1}{2}\theta e^{i(\varphi+\psi)/2}, \\ S_2&=(2S)^{\frac{1}{2}}\sin\tfrac{1}{2}\theta e^{i(\psi-\varphi)/2}. \end{aligned} \tag{16}$$

For the case where φ and ψ are both zero, these functions reduce to ordinary plane parabolic coordinates.[14]

We next have to show that these components are simply related to the components of certain vectors that we can associate with the spinor. Now there are various ways, geometrically, of associating a vector with a spinor. The simplest way is to let the vector have the same magnitude, direction, and sense as the spinor, as in Figs. 1 and 2; we shall call such a vector the associated longitudinal vector and denote it by **R**. Another way is to let the vector have the same magnitude as the spinor (that is, S), but make its direction the same as that of the spinor *blade;* we shall call this vector an associated transverse vector and denote it by **M**. Another transverse vector, **N**, of the same magnitude, can be drawn perpendicular to **M**. The arrangement of the three associated vectors is shown in Fig. 6.

The components and magnitude of **R** are related to the components of the spinor by the following easily verified equations (the asterisk

[11] Caution: the words *scalar*, *vector*, *tensor*, and the symbols i, j, k, originally introduced by Hamilton in his quaternion theory underwent some changes in meaning when they were later taken over into vector analysis.

[12] The application of matrices to the quaternion theory was first accomplished by Cayley in 1858. See Klein's book, reference 3.

[13] Birkhoff and MacLane, *A Survey of Modern Algebra* (Macmillan Company, New York, 1948), p. 212.

[14] See for example, Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), p. 54.

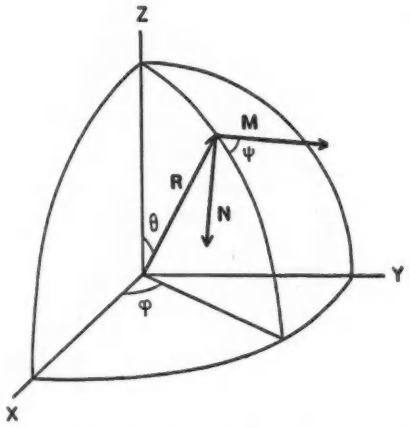

FIG. 6. The associated vectors.

denotes the complex conjugate),

$$\begin{aligned} R_x &= \tfrac{1}{2}(S_1S_2^* + S_2S_1^*) \\ R_y &= -\tfrac{1}{2}i(S_1S_2^* - S_2S_1^*) \\ R_z &= \tfrac{1}{2}(S_1S_1^* - S_2S_2^*) \\ R &= \tfrac{1}{2}(S_1S_1^* + S_2S_2^*). \end{aligned} \tag{17}$$

In the case of the transverse associated vectors, the relations are more complicated. We can compute the Cartesian components of **M** by first resolving **M** into a southward component ($M\cos\psi$) and an eastward component ($M\sin\psi$) on the surface of the sphere, then resolving each of these into its x, y, and z components and collecting results; we obtain

$$\begin{aligned} M_x &= S(\cos\theta\cos\varphi\cos\psi - \sin\varphi\sin\psi) \\ M_y &= S(\cos\theta\sin\varphi\cos\psi + \cos\varphi\sin\psi) \\ M_z &= -S\sin\theta\cos\psi. \end{aligned} \tag{18}$$

Similarly, we find for the components of **N**

$$\begin{aligned} N_x &= S(\cos\theta\cos\varphi\sin\psi + \sin\varphi\cos\psi) \\ N_y &= S(\cos\theta\sin\varphi\sin\psi - \cos\varphi\cos\psi) \\ N_z &= -S\sin\theta\sin\psi. \end{aligned} \tag{19}$$

Using Eqs. (16), (18), and (19), it is not difficult to verify that

$$\begin{aligned} M_x + iN_x &= \tfrac{1}{2}(S_1^2 - S_2^2) \\ M_y + iN_y &= -\tfrac{1}{2}i(S_1^2 + S_2^2) \\ M_z + iN_z &= -S_1S_2. \end{aligned} \tag{20}$$

We might call the left sides of Eqs. (20) the components of a *complex* vector **Q**; this complex vector is sometimes called a null vector (Kramers[1]) or an isotropic vector (Cartan[1]) because the sum of the squares of its three components is zero. (Kramers and Cartan develop spinor theory by first considering the associated vectors and then defining the components of a spinor by means of equations equivalent to our Eqs. (17) and (20).)

It should be noted that the vectors **N**, **M**, and **R** are arranged in exactly the same way as the electric vector, the magnetic vector, and the propagation vector in a plane progressive electromagnetic wave; also, **N** and **M** have equal magnitudes. This means that such a wave can be described by spinors without using vectors at all, since the specifications of **N**, **M**, and **R** are completely determined by those of the spinor **S**. However, any attempt to describe a *superposition* of electromagnetic waves in terms of spinors runs into trouble, because the rules for adding spinors do not run parallel to the rules for adding the associated vectors.

The study of the associated vectors may cause the reader to wonder why we take for the components of a spinor the complex, irrational expressions (16), instead of taking the components of two of the associated vectors, e.g., **R** and **M**, to be the components of the spinor, for after all the configuration consisting of **R** and **M** is geometrically equivalent to the spinor. This procedure would be uneconomical, because it would compel us to use a total of *six* components and two constraints or supplementary conditions (i.e., the conditions that **R** and **M** have equal magnitudes and perpendicular directions), whereas spinor theory based on the expressions (16) as components involves only four real (or two complex) quantities and no constraints at all.

8. ADDITION AND MULTIPLICATION OF SPINORS

Rules for adding and multiplying spinors can be set up in analogy with the familiar rules for vectors. To add two spinors we add their components separately, just as in the case of vectors. There appears not to be any simple geometrical construction analogous to the parallelogram of vectors for adding spinors graphically. Note that the sum of two antiparallel spinors is not zero; in order for the sum of two spinors to be zero they must both have the same magnitude, direction, sense, and angular position but be on opposite laps of the spinor manifold.

Before discussing spinor multiplication it will be useful to get a purely geometrical definition of a component of a spinor. This can be done by abstracting the purely geometrical content from the expression for S_1, Eqs. (16). This expression is a complex number whose absolute value is $(2S)^{\frac{1}{2}}\cos\frac{1}{2}\theta$ and whose phase angle is $\frac{1}{2}(\varphi+\psi)$. Now the expression $(\varphi+\psi)$ has a simple geometrical meaning; it is the angular separation that would exist between the blade of **S** and the blade of the basis spinor **u** if we were to rotate **S** in the plane of the handles of **S** and **u** until the two handles were brought into alignment. (This can be visualized with the aid of Fig. 7, in which **S** is shown as having the same magnitude as **u**, though this fact is not essential to the argument.) Therefore we shall call $(\varphi+\psi)$ simply the angle between the blades of **S** and **u**. The phase angle of S_1 is thus simply half the angle between the blades of **S** and **u**, the absolute value of S_1 being $(2S)^{\frac{1}{2}}$ times the cosine of half the angle between the handles of **S** and **u**. Generalizing these results, we can define the component of a given spinor referred to any arbitrary spinor as a complex number whose absolute value is equal to the magnitude function of the given spinor times the cosine of half the angle between the two handles and whose phase is half the angle between the two blades. This statement is the analog of the statement in vector analysis, that the component of a vector in any given direction is equal to the magnitude of the vector times the cosine of the angle between it and the given direction. Note that the angle between the blades should be measured from the blade of the reference spinor to that of the given spinor and is positive if this sense is counterclockwise. It is not hard to see, by imagining **S** rotated until its handle is aligned with that of **v**, that the above definition of a spinor component yields for the component of **S** referred to **v** just the second formula in Eq. (16), as it should.

We next define the *scalar product* of two spinors as the product of the magnitude function of the second by the component of the first referred to the second. According to the above definition of a spinor component, this is equal to the product of the two magnitude functions by the cosine of half the angle between the handles by a phase factor whose phase is half the angle between the blades. If the order of the two spinors is reversed in the scalar product, the sense of the angle between the blades will also be reversed, and the scalar product will therefore be the complex conjugate of the original scalar product.

The scalar product of two spinors **S** and **T** can also be gotten in terms of their components by the same process as is used in vector analysis, i.e., by writing $S_1\mathbf{u}+S_2\mathbf{v}$ for **S** and $T_1\mathbf{u}+T_2\mathbf{v}$ for **T** and multiplying these expressions term by term, noting that the scalar product of $(S_1\mathbf{u})$ and $(T_1\mathbf{u})$ in that order is by our definition $S_1T_1^*$ and that the scalar product of **u** and **v** is zero. The result for the scalar product of **S** and **T** is

$$S_1T_1^*+S_2T_2^*. \tag{21}$$

Two spinors whose scalar product is zero are called orthogonal. Since the scalar product contains as a factor the cosine of half the angle between the handles, the scalar product will be zero only if the spinor handles are antiparallel, the blades being arbitrarily directed. If two spinors are related to each other like **u** and **v**, that is, if the second is obtained by rotating the first through 180° in the plane and in the rotational sense indicated by the blade of the first, then the second is said to be the *dual* (or the spin-conjugate) of the first. The dual of **u** is **v**, but the dual of **v** is $-\mathbf{u}$, since the second 180° rotation carries us onto the second lap. If a spinor has components S_1 and S_2, the components of its dual are

$$-S_2^* \text{ and } S_1^*. \tag{22}$$

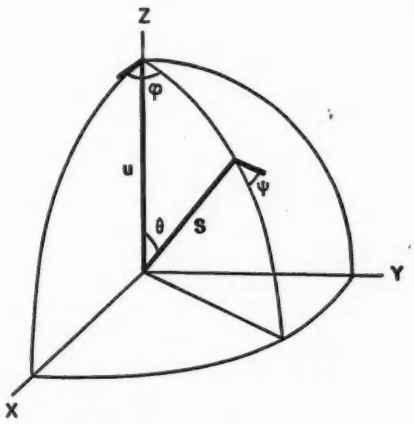

FIG. 7. The spinors **S** and **u**.

This result may be verified by identifying **S** with **u'** and the dual of **S** with **v'** in the matrix (9), and then noting the relations among the four quantities in the matrix (8). (In two-dimensional vector analysis the dual of a given vector **A** is obtained by rotating **A** 90° counterclockwise; its components are $-A_2$ and A_1.) The scalar product of one spinor **S** by the dual of another spinor **T** is equal to

$$S_2T_1-S_1T_2; \tag{23}$$

it is usually called the outer product of **S** and **T** and is the analog of the vector product of two vectors. Evidently (23) vanishes if **S** and **T** coincide.

There is another kind of product of two spinors **S** and **T**, the so-called direct product, defined as the aggregate of the four quantities

$$\begin{pmatrix} S_1T_1^* & S_1T_2^* \\ S_2T_1^* & S_2T_2^* \end{pmatrix}. \tag{24}$$

This is a *spinor of second rank*, but we shall not undertake to study the properties of such a spinor in this paper. If **S** and **T** are identical, then, according to Eqs. (17), (24) reduces to

$$\begin{pmatrix} R+R_z & R_x+iR_y \\ R_x-iR_y & R-R_z \end{pmatrix} \tag{25}$$

or

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}R+\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}R_x + \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}R_y+\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}R_z. \tag{26}$$

Note that the last three matrices in (26) are equal to $-i$ times the three matrices in (11).

If **T** is the dual of **S**, then (24) becomes, according to (20) and (22) and with $\mathbf{Q}=\mathbf{M}+i\mathbf{N}$,

$$\begin{pmatrix} Q_z & Q_x+iQ_y \\ Q_x-iQ_y & -Q_z \end{pmatrix}. \tag{27}$$

Since the magnitude of **Q** is zero, as mentioned just after Eqs. (20), the matrix (27) has exactly the same form as (25). Thus we can say that the direct product of a spinor by itself gives certain linear combinations of the magnitude and components of the associated longitudinal vector, and the direct product of a spinor by its dual gives the same combinations of the magnitude and components of the associated complex transverse vector.

The four quantities in (24) can be regarded as the coefficients of a linear transformation that can be applied to spinors. Linear transformations of the type (25) appear in relativistic quantum mechanics; however, we shall not undertake to deal with that subject in this paper.

In conclusion it seems appropriate to point out that the content of this article is only a fraction of all that is known about spinors. Further pursuit of the theory would lead us into the study of linear transformations other than rotations, spinors of second rank, and spinor functions of space and time, and we would also find it desirable to devote a good deal of attention to the physical applications of spinor theory. In this paper we have undertaken only to cover the basic part of the mathematical theory, and we have not attempted to discuss the applications in any detail.

I think mathematics has the advantage of teaching you the habit of thinking without passion. That seems to me the great merit of mathematics. You learn to use your mind primarily upon material where passion doesn't come in, and having trained it in that way you can then use it passionlessly upon matters about which you feel passionately. Then you're much more likely to come to true conclusions.—BERTRAND RUSSELL, Spinoza's Ethics (1942).

A Problem in Heat Conduction and Its Solution

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(Received October 15, 1951)

This paper gives a mathematical solution of a standard problem in one-dimensional heat conduction. It is intended to clarify the steps involved in solving certain boundary-value problems. Fourier series and integrals are involved, and the use of the probability integral is necessary for the final solution. Graphs of the original and final conditions are drawn to scale.

WHILE various methods may be used in solving problems in mathematical physics, many students have difficulty in understanding the processes involved in any method. It is for this reason that the following problem is given and the steps in the solution explained. It is hoped that this paper will be of assistance to both teacher and student.

THE PROBLEM STATED

A bar L centimeters long, whose sides are impervious to heat and whose ends have temperatures of 0°C and 100°C, is placed in contact between similar, very long bars at 0°C. The problem is to find the temperature at any point along the bar at any time after cooling begins.

After the solution to this problem is found, a specific application will be made to the case where the length of the bar is 100 cm and the diffusivity is 0.173. The temperature, fifteen minutes after cooling begins, will be calculated for several points along the bar.

Since this is a problem in one-dimensional heat conduction, it involves the solution of the partial differential equation[1]

$$\frac{\partial V}{\partial t}=a^2\frac{\partial^2 V}{\partial x^2}. \tag{1}$$

The following notations will be used:

V = temperature of bar at any point at any time and hence a function of both x and t.

x = coordinate point along the bar, measured from the left end where the temperature is zero, as shown in Fig. 1.

t = time in seconds, measured after cooling begins.

a^2 = a constant known as the diffusivity of the substance of the bar. It is equal to $k/c\rho$, where k is the conductivity, c is the specific heat, and ρ is the density. It is assumed that c, k, and ρ are independent of the temperature.

BOUNDARY CONDITIONS

As stated in the problem we see that

$$V=0, \text{ when } x<0 \text{ and when } x>L, \tag{2}$$

and

$$V=f(x)=(100/L)x \quad \text{when} \quad 0\leqq x\leqq L. \tag{3}$$

These original conditions are shown in Fig. 1.

SOLUTION

Let us assume that $V(x, t)=X(x)T(t)$, where X and T are functions of the single variables x and t, respectively. From this equation we have the

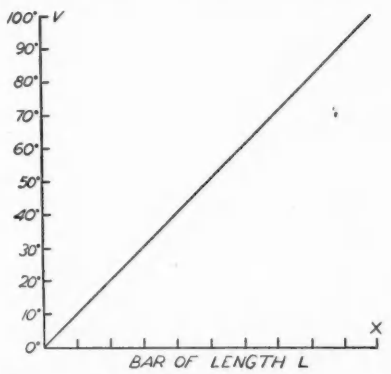

FIG. 1. Graph showing the original temperature distribution in the bar. The left end of the bar is taken as the origin of coordinates and the right end is at a temperature of 100°C as shown.

[1] James M. Cork, *Heat* (John Wiley and Sons, Inc., New York, 1942), p. 116.

following derivatives:

$$\partial V/\partial t = XT' \tag{4}$$

and

$$\partial^2 V/\partial x^2 = TX''. \tag{5}$$

Then from Eqs. (1), (4), and (5) we get

$$T'/a^2T = X''/X = -\lambda^2, \tag{6}$$

where λ^2 is a constant resulting from the fact that X and T are functions independent of one another. By integrating Eq. (6) we get

$$T = C \exp -a^2\lambda^2 t, \tag{7}$$

and

$$X = A_1 \cos\lambda x + B_1 \sin\lambda x, \tag{8}$$

where C, A_1, and B_1 are constants of integration.

Let us now assume that

$$V(x, t) = \int_{-\infty}^{\infty} [V_1(t) \cos\lambda x + V_2(t) \sin\lambda x] d\lambda \tag{9}$$

and determine $V_1 \cos\lambda x$ and $V_2 \sin\lambda x$ so as to satisfy Eq. (1) above. We find that

$$\frac{\partial V_1}{\partial t} \cos\lambda x = -a^2\lambda^2 V_1 \cos\lambda x \tag{10}$$

and

$$\frac{\partial V_2}{\partial t} \sin\lambda x = -a^2\lambda^2 \sin\lambda x. \tag{11}$$

Hence Eq. (1) is satisfied, and we have by integration

$$V_1 = A_2 \exp -a^2\lambda^2 t, \tag{12}$$

and

$$V_2 = B_2 \exp -a^2\lambda^2 t. \tag{13}$$

After substituting in Eq. (9) we get

$$V(x, t) = \int_{-\infty}^{\infty} \exp -a^2\lambda^2 t \times (A_2 \cos\lambda x + B_2 \sin\lambda x) d\lambda. \tag{14}$$

But when $t = 0$, we have $V = f(x)$, which can be expanded as a Fourier series

$$f(x) = \int_{-\infty}^{\infty} (f_1 \cos\lambda x + f_2 \sin\lambda x) d\lambda, \tag{15}$$

where by using α as a variable of integration we have

$$f_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\alpha) \cos\lambda\alpha d\alpha \tag{16}$$

and

$$f_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\alpha) \sin\lambda\alpha d\alpha. \tag{17}$$

As may be observed from Eqs. (3), (12), and (13), when $t = 0$, $V_1 = f_1 = A_2$, and $V_2 = f_2 = B_2$. Hence Eq. (14) becomes[2]

$$V(x, t) = \int_{-\infty}^{\infty} d\lambda \exp -a^2\lambda^2 t \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} f(\alpha) \times (\cos\lambda\alpha \cos\lambda x + \sin\lambda\alpha \sin\lambda x) d\alpha \right]$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} f(\alpha) d\alpha \times \int_{0}^{\infty} \exp -a^2\lambda^2 t \cos\lambda(\alpha - x) d\lambda. \tag{18}$$

To find the solution of the integral

$$I = \int_{0}^{\infty} \exp -a^2\lambda^2 t \cos\lambda(\alpha - x) d\lambda, \tag{19}$$

let $z = a\lambda\sqrt{t}$ and $A = (\alpha - x)/a\sqrt{t}$. From these equations we get

$$Az = \lambda(\alpha - x), \tag{20}$$

and

$$dz = a\sqrt{t} d\lambda. \tag{21}$$

Hence, we have

$$I = K(A)/a\sqrt{t}, \tag{22}$$

where

$$K(A) = \int_{0}^{\infty} \exp -z^2 \cos Az dz. \tag{23}$$

If we differentiate under the integral sign with respect to A, we get

$$\frac{dK}{dA} = -\int_{0}^{\infty} \exp -z^2 z \sin Az dz, \tag{24}$$

and integrating this by parts we find that

$$\frac{dK}{dA} = -\frac{1}{2} \exp -z^2 \Big|_0^{\infty} - \frac{A}{2} \int_{0}^{\infty} \exp -z^2 \cos Az dz$$

$$= -\tfrac{1}{2} AK, \tag{25}$$

[2] Burington and Torrance, *Higher Mathematics* (McGraw-Hill Book Company, Inc., New York, 1939), pp. 548–550.

since the first term on the right vanishes. From this we find that

$$K = K_0 \exp -\tfrac{1}{4}A^2 = K_0 \exp[(\alpha - x)^2/4a^2t], \quad (26)$$

where by letting $A = 0$ in Eq. (23) above we have

$$K_0 = \int_0^\infty \exp -z^2 dz = \frac{\sqrt{\pi}}{2}. \quad (27)$$

Substituting this in Eq. (22) we get

$$I = \tfrac{1}{2}(\sqrt{\pi}/a\sqrt{t}) \exp[(\alpha - x)^2/4a^2t], \quad (28)$$

and hence, upon simplification Eq. (18) becomes

$$V(x, t) = (1/2a\sqrt{\pi t}) \times \int_{-\infty}^{\infty} f(\alpha)\exp[(\alpha - x)^2/4a^2t]d\alpha. \quad (29)$$

Now if we let $y = (\alpha - x)/2a\sqrt{t}$, we find that

$$\alpha = x + 2a\sqrt{t}y, \quad (30)$$

and its differential is

$$d\alpha = 2a\sqrt{t}dy. \quad (31)$$

Therefore,

$$V(x, t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x + 2a\sqrt{t}y) \exp -y^2 dy. \quad (32)$$

The development thus far may be found in several textbooks.[3]

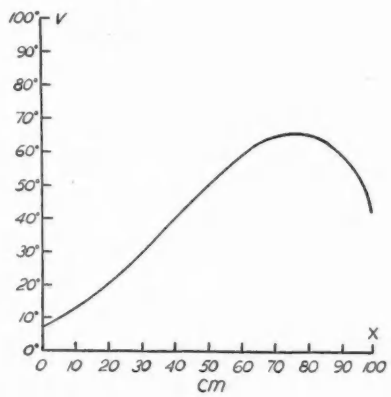

FIG. 2. Graph showing the final temperature distribution in the bar after fifteen minutes of cooling. The units used are the same as those assumed in Fig. 1 in order that comparisons may be made.

[3] Carslaw and Jaeger, *Conduction of Heat in Solids* (Oxford University Press, New York, 1947), pp. 33–34.

But from Eq. (30) we find that when $\alpha = 0$, $y_1 = -x/2a\sqrt{t}$, and likewise $f(\alpha) = 0$ as Eq. (3) reveals. We also observe that $y_2 = (L - x)/2a\sqrt{t}$, when $\alpha = L$, so that the final solution of the problem is

$$\begin{aligned} V(x, t) &= \frac{1}{\sqrt{\pi}} \int_{y_1}^{y_2} \frac{100}{L}(x + 2a\sqrt{t}y) \exp -y^2 dy \\ &= \frac{1}{\sqrt{\pi}}\left[\frac{100}{L}\int_{y_1}^{y_2} x \exp -y^2 dy \right. \\ &\qquad \left. + \frac{200a\sqrt{t}}{L}\int_{y_1}^{y_2} y \exp -y^2 dy\right] \\ &= \frac{100}{L\sqrt{\pi}}\left[x\int_0^{y_2} \exp -y^2 dy \right. \\ &\qquad - x\int_0^{y_1} \exp -y^2 dy \\ &\qquad \left. - a\sqrt{t}(\exp -y_2^2 - \exp -y_1^2)\right]. \quad (33) \end{aligned}$$

If we now apply this formula to the case where $L = 100$ cm, $t = 900$ sec, and $a^2 = 0.173$,[4] we have $2a\sqrt{t} = 25$. Furthermore, for the following points along the bar: $x = 0$, 25 cm, 50 cm, 75 cm, or 100 cm, we have, respectively,

$$y_1 = 0, \ -1, \ -2, \ -3, \text{ and } -4,$$

and

$$y_2 = 4, \ 3, \ 2, \ 1, \text{ and } 0.$$

Likewise, from tables of probability integrals we find that

$$\frac{2}{\sqrt{\pi}}\int_0^1 \exp -y^2 dy = 0.84270, \quad (34)$$

$$\frac{2}{\sqrt{\pi}}\int_0^2 \exp -y^2 dy = 0.99532, \quad (35)$$

$$\frac{2}{\sqrt{\pi}}\int_0^3 \exp -y^2 dy = 0.99998, \quad (36)$$

and

$$\frac{2}{\sqrt{\pi}}\int_0^4 \exp -y^2 dy = 0.99999. \quad (37)$$

[4] This value of 0.173 for the diffusivity is of the right order of magnitude but is not applicable to any common material.

Therefore, at the points along the bar given above, the temperature readings are as follows:

$$V_0 = -\frac{a\sqrt{t}}{\sqrt{\pi}}(e^{-16} - e^0) = 7.05°\text{C}, \quad (38)$$

$$V_{25} = \frac{1}{\sqrt{\pi}}\left[25\int_0^3 \exp -y^2 dy + 25\int_0^1 \exp -y^2 dy - 12.5(e^{-9} - e^{-1})\right] = 25.6°\text{C}, \quad (39)$$

$$V_{50} = \frac{1}{\sqrt{\pi}}\left[100\int_0^2 \exp -y^2 dy\right] = 49.77°\text{C}, \quad (40)$$

$$V_{75} = \frac{1}{\sqrt{\pi}}\left[75\int_0^1 \exp -y^2 dy + 75\int_0^3 \exp -y^2 dy - 12.5(e^{-1} - e^{-9})\right] = 66.5°\text{C}, \quad (41)$$

and

$$V_{100} = \frac{1}{\sqrt{\pi}}\left[100\int_0^4 \exp -y^2 dy - 12.5(e^0 - e^{-16})\right] = 42.9°\text{C}. \quad (42)$$

The temperatures at other points not given were calculated in order that an accurate graph could be plotted (Fig. 2). The student may be interested in the way the bars at the ends affect the temperature of the bar in question, as shown by the form of the curve.

The author expresses his thanks to Doctor C. E. Demaray, Chairman of the Division of Languages and Literature, and to Professor Winona L. Kell, Assistant Professor of English, for their helpful suggestions on the manuscript.

In this we must, of course, take shape (or Gestalt) in a much wider sense than as geometrical shape. Indeed there is no observation concerned with the geometrical shape of a particle or even of an atom. It is true that in thinking about the atom, in drafting theories to meet the observed facts, we do very often draw geometrical pictures on the black-board, or on a piece of paper, or more often just only in our mind, the details of the picture being given by a mathematical formula with much greater precision and in a much handier fashion than pencil or pen could ever give. That is true. But the geometrical shapes displayed in these pictures are not anything that could be directly observed in the real atoms. The pictures are only a mental help, a tool of thought, an intermediary means, from which to deduce, out of the results of experiments that have been made, a reasonable expectation about the results of new experiments that we are planning. We plan them for the purpose of seeing whether they confirm the expectations—thus whether the expectations were reasonable, and thus whether the pictures or models we use are adequate. Notice that we prefer to say adequate, not true. For in order that a description be capable of being true, it must be capable of being compared directly with the actual facts. That is usually not the case with our models.

But we do use them, as I said, to deduce observable features from them. It is these that constitute the permanent shape or form or organization of the material object, and they have usually nothing to do with 'tiny specks of material, constituting the object'.—ERWIN SCHRÖDINGER, Science and Humanism (1951).

A. A. Knowlton

Recipient of the 1951 Oersted Medal for Notable Contributions to the Teaching of Physics

The American Association of Physics Teachers has made to A. A. Knowlton, Emeritus-Professor of Physics at Reed College, the sixteenth of its annual awards for notable contributions to the teaching of physics. The address of recommendation printed below was made by Professor Duane Roller, Chairman of the Committee on Awards, and the presentation of the medal and certificate was made by Professor Mark W. Zemansky, President of the Association, at a ceremony in McMillan Theater, Columbia University, New York, on February 1, 1952, during the twenty-first annual meeting.

The Oersted Medal and the Significance of Its Award to Professor A. A. Knowlton

DUANE ROLLER
Chairman of the Committee on Awards for 1951

THE annual award for notable contributions to the teaching of physics, which goes this year to PROFESSOR A. A. KNOWLTON, has a history that now covers a period of some 17 years. Although the details of this history have been recorded through the years in the *American Journal of Physics*, the early parts may not be well known to many of the present members of the Association, for the majority of them have joined since the time when the plans for the award were developed.

The first suggestion for such an award was made in 1934, during the annual meeting of the Association in Pittsburgh. A planning committee was appointed and, at the next annual meeting, in St. Louis, it presented a definition of the character of the proposed award and a set of principles for selecting recipients. The plans were approved, and a Committee on Awards was instituted. This committee, for any particular year, now consists of the president of the Association, the two most recent past presidents, the secretary, and the medalist of the preceding year.

The first of the annual awards was made during the annual meeting at Atlantic City in 1936. In an address made at that time, the president of the Association, who was then D. L. WEBSTER,[1] pointed out that this award was not only new, but of a new kind; that for teaching, unlike research, there had in the past been relatively few special honors and prizes.

The reason for this difference is primarily historical, said Webster. Teaching has gone on for centuries and has long been recognized as a necessary art, essential to civilization and therefore laudable—so laudable, in fact, that the

[1] D. L. Webster, Am. J. Phys. (Am. Phys. Teacher) **5**, 31 (1937).

teacher, like the minister, was assumed to be rewarded so much by the satisfaction of doing his best for such a cause that he needed little of the other awards. Meanwhile, several generations ago in this country, research was an idiosyncracy, and the few research workers we had were not understood. Then came the realization that research was also laudable. Time was allowed for it in a professor's schedule, and money was allowed for apparatus. The stimulation of research was carried out by every means, including honors.

But if such awards and honors serve to stimulate research, then would they not also stimulate more physicists to engage in productive teaching? It was therefore with great satisfaction, as Professor Webster said, that we could view the establishment of an annual award for notable contributions to the teaching of physics.

> [It has never been the intention] to make this award an object of competition. Whatever may or may not be said of prizes for research and of the difficulty of judging the relative merits of researches in different fields, far more may be said of the difficulty of judging teachers; human judgment is too fallible in such matters for any man or any committee to presume to judge such a competition.
>
> It would be even more inappropriate for any person receiving an award in this field to consider himself thereby designated as the winner of such a competition. On the contrary, the recipient of this award is designated as a representative of an ideal—an ideal of the great teacher. The purpose of the award is to hold this ideal before us, to remind us of a teacher who has gone far toward it, and to say to each one of us, "Go, thou, and do likewise."[1]

It was this underlying philosophy, described so ably in connection with the first award, that guided the present Committee in its selection of Professor Knowlton as the Oersted medalist for 1951. When the Committee first made its recommendation known, in December, the typical reaction of a number of people was: "*Knowlton?* I thought he received the medal a long time ago. He should have!" What he did receive several years ago was a citation for his educational contributions to physics from the Research Corporation, a citation that was all the more remarkable because it involved a cash award of a thousand dollars. The idea of a teaching award that, 17 years ago, was new, and even considered bold, has spread and developed.

Professor Knowlton was a country boy, and probably still is, at heart. Back around 1886, when he was 10 years old, he is said to have announced to his widowed mother that he had made up his mind about his future: he was going to be a teacher. Most of his early boyhood was spent on a farm some 100 miles north of Portland, Maine. There he attended "the little red schoolhouse" and, to quote him, "thus escaped all graded schools and pedagogically trained teachers until high school age."

Of his later school experience, at Anson Academy, in Maine, he has said:

> Here I encountered one of the relatively few people who contributed something vital to my attitudes and philosophy. Miss B— was a cranky old maid with small sympathy for negligent students and no taint of any soft pedagogy. Any attempt to stall in her classes was greeted with: "Come now, either you know it or you don't. Which is it?" I tremble to think of the brevity of her probable tenure in any modern school system. She was not much of a Latin scholar, but I think she did something outstanding in the way of stimulation and character development.

Knowlton says that his mother always had assumed, as a matter of course, that he was to go to college; otherwise he probably never would have embarked upon what, in that period, was an ambitious undertaking for one in his economic situation. Nearby was Bates College, and from there he graduated in 1898. The general emphasis at the time was on the classics, and his physics training at Bates was confined to one course, part of which he missed while away teaching country school to make enough money to continue in college.

After graduation, he taught for a year in a Massachusetts high school and then went to Carleton College, in Minnesota, which had been "looking for a man of many talents who could coach its athletic teams with one hand and teach science with the other." Carleton had several outstanding men on the science faculty, and it was also there that he met the young woman, Gertrude Griffin, who was to become his helpmate for some 40 years.

After three years at Carleton, he was awarded an assistantship in physics at Northwestern University, studying under Professor Henry Crew, who, Knowlton has said, "so broadened and deepened my understanding of physics that

I have often charged him with being my intellectual godfather."

With the Master's degree completed, Knowlton went to Armour Institute of Technology as an instructor. During his six years of tenure there, and while teaching a full-time program, he completed work for the doctorate at the University of Chicago, receiving the degree in 1910. In the whole period between the A.B. and the Ph.D., Knowlton regards himself as having been "lucky beyond all expectation" in the men with whom he came into contact: the scientists at Carleton; Crew at Northwestern; Michelson, Millikan, Gale at Chicago. It is of interest that two of these former teachers of his, Crew and Millikan, are also Oersted medalists.

In 1909 Knowlton went to the University of Utah. In 1915 Reed College was looking for a new physics man; for, from its original faculty of four years earlier it had just lost K. T. Compton. Knowlton was invited to take Compton's place. And at Reed he remained for 33 years, until his retirement in 1948. Afterwards, he taught for two years at Bennington College, back in the general region where he had spent his boyhood, and then he re-retired.

The Reed College physics department under Knowlton had a remarkable record. One study, made by Blackwood,[2] of the undergraduate origins of American physicists rates Reed College almost twice as high as any other institution. Both the Steelman Report to President Truman and the study recently made by Knapp and Goodrich[3] give Reed superior ratings in the development of scientists, and the physics contribution to this record is a large one. During Knowlton's tenure at Reed, 113 students graduated with a major in physics and, of these, at least 35 percent entered the profession of physics, while another 20 percent pursued some branch of engineering or other work requiring a background of physics.

Some of Knowlton's former students have said that there was nothing extraordinary about his performance as a classroom teacher. However, this seems to be just one of a number of instances that can be cited, where a professor rated by his students as average in classroom performance has nevertheless sent many of them on to successful careers in physics. Is it possible that a popular, somewhat emotional sort of classroom manner, while of value in instructing the general public, is less effective in attracting into physics those students having the aptitudes necessary for the sciences; indeed, that it also may not be very effective in helping students to develop the character traits and attitudes most essential for a successful scientific career?

Obviously Knowlton has had the ability to foster these requisite traits and attitudes in students. We also know that the atmosphere in his department was such as to encourage creativeness in students. Certainly, and in the first place, he could spot and encourage those young people in his elementary course who had the qualities needed for success in physics.

But before such vocational guidance is possible, a physics department in a liberal arts college must find ways to attract students into its first-year course. Why Knowlton's first-year course was efficacious in this respect should be clear to anyone familiar with the first edition of his *Physics for College Students*.[4] This textbook is bold in conception and contains many innovations. It shows that in his elementary course he treated physics from a humanistic standpoint, yet with no sacrifice of technical values. He centered the treatment on a few broad problems, each with the concept of energy as the central theme, thus disregarding the traditional divisions into mechanics, heat, electricity, and so on.[5] He utilized a pedagogic device that he has called the "soaking in process": no fundamental topic is treated fully when first introduced, only so much being done with it as is required for use at the point; later, more is added, but now to material that has already become a part of the student's mental equipment through use. Lastly, his course contained much "modern physics," thoroughly integrated with the classical material, instead of being put at the end of the course; and these additions were offset, to advantage,

[2] O. Blackwood, Am. J. Phys. **12**, 149 (1944).

[3] R. H. Knapp and H. B. Goodrich, Science **113**, 543 (1951); H. B. Goodrich, *et al.*, Sci. Am. **185**, 15 (1951).

[4] McGraw-Hill Book Company, Inc., New York, 1928.

[5] In the second edition (1935), Knowlton reorganized his material into the traditional divisions, as a concession to the general demand for a textbook so divided that a first-year physics student can, if he desires, painlessly transfer in midyear from one college to another.

by eliminating material that is less fundamental or that can better be treated through lecture demonstration or in the laboratory.[6]

As was emphasized earlier, the purpose of this award is to set up an ideal. To paraphrase something once said by the late Professor William James, if any young physicist who wishes to teach will set out faithfully, day by day, to emulate the kind of example furnished by Professor Knowlton's career, he can safely count on waking up some fine morning to find that he, too, has become one of the competent teachers of his generation.

[6] For an extensive discussion of the "new physics" and the first-year course, see Knowlton's article in Am. J. Phys. 4, 71 (1936).

Freedom in Research in a University

Freedom in research is an expression of man's right to liberty in thought, inquiry and communication. The university, traditional stronghold of freedom and perhaps its last defense, is particularly concerned with this issue as an ideal, as a basis for practical procedure, and as an instance of academic freedom. Independence of action in research has significance for the university as an institution, and for the faculty and graduate students as individuals.

With war clouds on the horizon, it may be thought that freedom in research could be safely shelved for the present. However, it is just under such circumstances that our ideal of scientific and academic freedom must be reaffirmed and practiced, lest through misapprehension and misdeeds we sacrifice our ideal while attempting to retain our liberties.

The broad principle of freedom for mankind is increasingly contradicted by the trend to restrict independence in investigation and communication of results of research. Secrecy is justifiably applied to research done under academic circumstances when a national emergency exists and the disclosure of the results of a particular piece of research would clearly jeopardize national security. However, this restriction is so contrary to the broader interests of society that it should be applied only when there is a genuine national emergency and the evidence obtained after careful scrutiny of the research in question clearly makes secrecy mandatory. Sequestration of newfound information acquired in the university under support of industry may seem valid on the basis that industry may be entitled to exclusive rights to the results of research it sponsors. There are, however, broader considerations which govern the degree of acceptability of this practice.

The institution's relation to the principle and practice of freedom in research has to do basically with the general atmosphere of a university, the agreements with outside agencies it makes and approves, its provision of facilities, and its acceptance of obligations incurred. In spite of certain conditions apparently justifying restrictions and secrecy in research, the issue remains as to whether research so conducted is compatible with the spirit and responsibilities of a university. Gaining new knowledge is a mark and an obligation of a university. How investigations are conducted and what is done with the results for the benefit of society are also the concern of a university. . . .

Freedom in research is to be guarded also in connection with employment of students. Without respect to secrecy, the graduate student should avoid accepting appointment as a Fellow to work on a problem which diverts him from his main interest and plans. He loses his freedom when the fellowship is so restricted in its terms that he is required to follow a course unsuited either for his ability or his independence of thought. The student should be protected in his freedom and directed to a more valid course.

A university has its responsibility to the student as well as an obligation to add to knowledge. Let us not put our students in the position of observing or being a party to practices contrary to the ideal for a university and for his future career. If the student sees his institution engaged in fundamental research, freely supported and freely conducted, he will learn that independence in investigation is man's right and a university's responsibility to exemplify.

The factors adversely affecting academic freedom and inducing loss of independence by the university, faculty or students are obvious: financial gain, self-importance, lack of an original plan of action, weakness in declaration of principles, and indifference. When these factors determine the course of action, the university as an institution and as a collection of individuals has lost its independence, leadership and strength.

Freedom in research is an expression of the spirit and atmosphere of a university. Neglected, it disappears; actively sought, it remains. Whether in war or peace, the university should vigorously stand for the principle of freedom in research, discouraging secrecy and encouraging free discussion and transmission of the fruits of study. On these principles, universities will endure.—N. Paul Hudson, Ohio State University Graduate School Record, December, 1951.

Opportunities and Rewards in Physics Teaching

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(Received January 11, 1952)

Response to the presentation of the Oersted Medal of the American Association of Physics Teachers, February 1, 1952. The article is directed particularly to young physicists and is intended to present some of the more attractive features of small college teaching for their consideration.

PERHAPS I should begin with two comments in limitation and clarification of my title. In the first place, it will be readily understood that my remarks are intended particularly for the consideration of the younger men in this distinguished audience. In the second place, I shall speak particularly about undergraduate teaching in the smaller liberal arts colleges as distinguished from the great universities on the one hand and technical schools on the other. I hope that no one will ask me to define precisely what I mean by a "small college." In fact, such a question does not admit of an answer in any general terms, but I have complete confidence that none of you will have any difficulty in making the proper classification when and if the necessity arises.

My selection of this topic was largely motivated by the rather considerable number of articles lamenting the sorry state of the college teacher which have appeared in educational and popular publications during the past few years. It is true that so far as I can recall, none of these articles have come from teachers of physics in such institutions as I have in mind. It may therefore seem that I am tilting at a windmill,—an activity which has for some two centuries or more been regarded as the acme of futility. I hope that this is right. I hope that it is, in fact, needless to say anything by way of encouraging young men of the highest ability to consider undergraduate teaching in the smaller colleges as a life profession. But I recall that one author wrote under the title: "College Teaching: The Loneliest Profession."[1] The concluding paragraph of this article summarizes the defeatist attitude found in many others:

> To the man or woman who is really interested in teaching, American colleges now offer a very meager career. He will have, in the response of his students, a kind of personal reward every day; but he must resign himself to a life of obscurity and frustration, living not in an ivory tower but in a dingy side street. For him teaching will be the loneliest profession in the world.

As a matter of fact, a careful reading of the article does not seem to justify so pessimistic a conclusion. The thesis developed is a rather familiar one. It is an attack in force along three lines: (i) College instructors do not know how to teach and are too uninterested to improve their teaching. (ii) College administrators know little and perhaps care less about the classroom performance of the instructor so that advancement is dependent upon quantity of publication. (iii) the graduate schools overstress the importance of research and do nothing to encourage teaching ability.

It is not my purpose to attempt any extended discussion of these charges. I am a graduate of a small college and have spent nearly forty years as a teacher in such institutions. I have seen good teachers and teachers who have grown better with the passing years. I have also seen mediocre and inefficient teachers as well as teachers who failed to grow better as the years passed. It is my experience that the quality of instruction in such small colleges is upon the whole good and that the performance of every instructor is continuously evaluated and well known to students, fellow faculty members, and administrators.

So far as the graduate schools are concerned, and please remember that I am speaking only of the departments of physics, I have no disposition to rush to the defense of either their curricula or methods. This is because I think that no such defense is necessary. Of course, their primary duty is and must continue to be the training of scholarly and productive workers in a highly specialized field. Perhaps if my subject included a

[1] Am. Assoc. Univ. Prof., Bulletin, **35**, No. 42, 643 (1949).

consideration of undergraduate instruction in the very large institutions, I might have some comments to make, not only as to the criteria for professional advancement, but also as to other problems having to do with mass education.

It is my frankly expressed purpose to paint a more attractive picture of college physics teaching than that suggested by such articles as the one mentioned earlier. I am quite well aware that every young Ph.D. in physics who is worth his salt is interested in research. I must then consider how research and teaching may supplement each other in the small college.

It has been said that the habit of indulgence in reminiscence is a sign of senility, and I find myself increasingly sensitive upon this point. Nevertheless, I shall begin by recalling some matters familiar to all the older men here present but largely incomprehensible to the younger group.

Three of the five major discoveries which initiated the period of modern physics as distinguished from the so-called classical period were made while I was an undergraduate. These were: discovery of x-rays in 1895, Becquerel rays in 1896, and proof of the existence of the electron in 1897. The two great theoretical steps necessary for the exploitation of these experimental results came in the early years of the present century. You are all familiar with the great increase in fundamental knowledge and the myriads of industrial applications which have followed and are continuing to follow the amplification of these discoveries. But the younger group may not be equally familiar with the way in which these developments have affected the physicist himself as a professional man.

In the early years of the century and for a considerable time thereafter, almost the only gainful employment open to the young Ph.D. in physics was that of teaching. If he had an unusually brilliant record, he might be fortunate enough to find a position in which research activities could accompany his duties as an instructor. Such opportunities were for the most part to be had only in the larger universities.

As an example of the attitude then existing in at least some of the smaller colleges, I may cite a case in which the president of one such institution of good repute offered me a position but made it a condition that I should abandon the completion of my work for the doctorate, as well as any further attempt to carry on research. I suspect that the good gentleman never really understood me when I attempted to explain a belief, which has grown stronger with the passing years. To wit: The belief that at least a minimum of research activity is necessary if one is long to remain a vital and stimulating teacher of undergraduate physics. Even then I had seen too many cases in which the content of a course in general physics at its completion must have seemed to the student somewhat like the contents of an art gallery or museum; that is, like an orderly array of pictures and artifacts all completed, framed, and hung upon the wall or stored in cases safely locked against meddlesome hands. My idea, which I tried to explain to this president, was that such a course ought to appear as a series of glimpses into studios and workshops in which artists had worked and were still working upon objects of interest which always remained in some respects unfinished.

Small wonder that the good man escaped as quickly and as tactfully as possible, trailing behind an unfinished "If you should change your mind. . . . " The imagined disorder of such a course must have shocked him beyond expression.

Today, you will find that most if not all college administrators will welcome your desire to continue research and will, within the limits of their means, be helpful. Within their means—that is the catch. Carrying on physical research requires a place in which to work, at least a minimum of apparatus, library facilities, and time. Of these, time is at present perhaps the most difficult to wrangle from the administrator. Yet the instructor can circumvent most of these difficulties by exercising the same ingenuity and patience which are necessary for carrying on the research itself.

Just at present, small colleges in the greater part of the country are being severely pinched financially. Low interest rates have cut into the returns from endowments and inflation has increased all costs. In spite of being thus caught between hammer and anvil, these institutions have managed to survive and even to increase salary scales, along with other expenditures. I am speaking of this because such things are basically important to one considering employment. No

one would think it good judgment to put out to sea for a prolonged voyage in an unseaworthy craft.

Being a confirmed optimist and having had considerable opportunity in the past to observe both the vitality and the resilience of small colleges, I have great confidence in their future.

Strangely enough, in the light of what has just been said, the problem of securing funds for research is, for the instructors in the small college, easier than ever before. This paradoxical situation arises, of course, from the increased appreciation of the value of physical research which is one of the by-products of the last war. In particular, it has arisen from the discovery by government agencies and by the various research foundations that there are a considerable number of competent physicists in the small colleges who can be made productive if supplied with the necessary financial aid. There is, in addition, the rather recent realization of the fact that these small colleges furnish a large fraction of the scientific neophytes—those peculiar and highly selected individuals who become professional scientists.

The creation of a research atmosphere in a small college where it does not already exist is not the work of a day. In fact, this task is itself a research problem fit to challenge the best efforts of any instructor. In general, it will not develop simply from the fact that the instructor is himself working in some fenced-in area, with apparatus which no one is permitted to touch, upon some problem about which the students know nothing. It must be that in some way the student himself becomes a participant. All research arises out of attempts to satisfy curiosity. An unanswered question is the starting point. Sometimes, a suggestion that he seek a solution himself is all that is necessary. I recall a day away back, when a stocky red-haired lad stopped after a session in which we had been discussing the Michelson interferometer and startled me with the question: "Could I make an interferometer?" I really had no faith in the possibility, but instead of a flat *No*, I said: "Think it over, see what you can find to work with and try it if it seems worthwhile." Quite to my surprise, he appeared a few days later with a gadget concocted from bits of plate glass, a support, and a micrometer screw. It wasn't an instrument of precision, but with sodium light it gave fringes which moved when the screw was turned. I still think there must have been something screwy about it, but that day a physicist was born.

It is one of the greatest opportunities of the small college instructor that he need not confine the laboratory work to that which appears in the manual, nor insist upon the completion of a laboratory exercise (these things are not properly called experiments) each period. I well remember the shock which an able and hard-headed youngster gave me some forty years ago when he objected strenuously to doing a certain routine "experiment," so called. "Why," he asked, "should I spend three hours on that? I understand the theory of it and I already know the value of the acceleration due to gravity more accurately than I am likely to get it with that rig." Since I knew, in addition, that the lad was a careful and competent manipulator of apparatus, I found no satisfactory answer to his question; so I dodged by saying: "There's a lot of apparatus around here. Go and find some job which you do think worthwhile." I don't know whether he learned anything that afternoon or not—but I did.

Such opportunities for the treatment of students as individuals are among the rewards of small college teaching. Perhaps I ought to warn you that this kind of teaching is time consuming and far more exacting than ordinary classroom routine. I have pointed out that the opportunities for research activity in the small colleges are likely to be just about what a man makes them. One warning, I think, should be given. The small college physicist must not put too high a value upon publication. To be sure, publication is the desirable culmination of research. But for the small college teacher, creation of the research spirit, if it does not already exist in the particular institution, must be the primary objective. Once this goal has been reached, he will find himself surrounded by a group of competent and enthusiastic helpers to whom fragments of larger problems upon which he is working may be farmed out to the profit of the student, the instructor, and the college.

Some results of the recent investigations of Goodrich, Knapp, and Boehm into the "Origins

of the United States Scientists" are to be found in their article under this title in the July, 1951 issue of *The Scientific American.*

Two brief quotations from this have a bearing upon this discussion,

> the small liberal arts colleges are by far the most productive sources of future scientists among U. S. institutions.

(Productivity is here measured in terms of the proportion of graduates who become scientists.)

At another point they say

> Certainly the clear demonstration of the contributions of the smaller liberal arts colleges to the scientific profession should be of considerable interest to those formulating our national policies.

The December, 1951 issue of *Physics Today* carries (page 29) an article under the title, "Scientific Manpower; The Shortage is Becoming Critical." This article contains a quotation from Defense Mobilization Director Charles E. Wilson, from which I quote the following:

> The supply of scientific and engineering graduates in 1951 is less than half that needed to fully man our economic and defense programs.

Among other suggested means of alleviating this situation, he suggests

> Our educational institutions can develop counseling programs which will result in a larger number of men and women being trained in these fields. Students with aptitudes for such training can and should be shown that by obtaining scientific and technical training they can prepare themselves for satisfying employment and at the same time make a major contribution to the preservation of our way of life.

Later he says,

> I appeal to our educational institutions . . . to take every available measure to help alleviate the shortage of scientific and technical personnel.

Certainly in the light of this and the Knapp report, the maintenance of a high standard of physics teaching in small arts colleges may be regarded as an opportunity for an important contribution to national defense.

Doubtless you have all visited the employment service rooms where you found representatives of industrial and government laboratories as well as of educational institutions lying in wait for you. I do not know what the situation is here, but it is said that at the Chicago meetings a few months ago, the ratio of jobs to job hunters was something like four to one. I congratulate you upon having this kind of a market for your wares. Furthermore, let me assure you that whatever choice you may make, after a careful investigation and weighing of the opportunities in each field in the light of your own tastes and abilities, will be a good choice. By this I mean that you will probably find yourself moving along a road leading to success and satisfaction. If you like teaching, it is probable that the more intangible factors will be sufficient to outweigh the lesser financial returns. If it continues to be true that future scientists come in large numbers from small colleges, it will be because a number of well-trained young men of ability and character each year choose to enter the profession of teaching in such institutions.

May I add a personal word in conclusion. To me it appears that the most intriguing opportunities and the greatest rewards for the physicist in the small college lie in the freedom which he has as a teacher. The course in general physics is the critical one. So far as this course is concerned, there are only two requirements which must be met. It must appeal to the student as being worthwhile and it must hold his interest. If these two objectives are not reached, this course will almost certainly wither away and with it will go all chance of developing more advanced work and a research atmosphere. Here is a challenge fit to call out any man's best efforts. I am inclined to believe that these two conditions may be reduced to one. It is my experience that a sustained and growing interest in any field develops as an accompaniment of conscious progressive achievement in that field. The material of general physics is to be found in any textbook. The sense of continuity, the plot, if you will, must be supplied by the instructor. In doing this he will be completely free in the small college to try any class room and laboratory methods, as well as to adopt any choice, ordering, and treatment of material which seems likely to contribute to the desired end. These institutions are the ones in which radical experiments in teaching are most easily undertaken and most likely to succeed. Finally, educational experimentation of this sort can be great fun for both instructor and students. I recommend it for your consideration.

The Prismatic Curvature of Spectral Lines

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(Received December 14, 1951)

A review of the early papers on the curvature of spectral lines is presented. The formulas for the oblique refraction of a light ray incident upon a boundary between two media are derived by means of Fermat's principle. These results are applied to the prismatic refraction of the oblique rays that originate at the off-axis positions of the spectrometer entrance slit. Three algebraic forms for presenting the resultant curvature of the slit image are given. Graphical results are presented relating the radius of curvature with the index of refraction of the prism at constant prism angles. Comparison is made with the curvature caused by reflection from a plane grating.

THE curvature of the image of a spectrometer slit by a prism appears not to have been extensively discussed in modern literature. The determination of the curvature is of the greatest importance in the design of infrared monochromators. The effect is compensated for either by matching a curved exit slit to the prismatic curvature of the entrance slit image or by forming a curved entrance slit designed to result in straight line images. Generally, the latter is preferred because of the narrow linear form of most infrared detectors. It is interesting from a historical standpoint to note that Coblentz[1] mentions slit image curvature as resulting in a definite calibration discrepancy between two early infrared prism spectrometers used in the first mapping of the absorption bands of organic compounds. Correction is not necessary for work in the photographic region of the spectrum, unless only a portion of the line above or below the optical axis is being made. Then for accurate wavelength calibration the proper shift resulting from curvature will need to be calculated.

Recent spectroscopy texts quote only the resultant formula for the radius of curvature of the distorted spectral lines, so that the student of the subject has little idea concerning the details of the derivation or the accuracy of the result. The derivation given by Kayser,[2] although complete, contains an error in the final equation. Schaefer and Matossi[3] quote from Subrahmaniam and Gunniaya[4] to present the curvature in terms of the angles of deviation without further explanation. The Perkin-Elmer manual for the infrared spectrometer, Model 12C, gives the curvature in terms of the sagitta of the curve. For this reason it was thought appropriate to review the derivation and to relate the different expressions here.

Perhaps the greatest obstacle to the student of spectroscopy is that we are concerned in this problem with the rays traversing the prism obliquely rather than simply in the principal section as considered in most optics texts. Only the rays from the center of the slit will be collimated to pass through the prism in a principal section. Those from the ends of the slit parallel to the refracting edge will be collimated obliquely to the principal section. Therefore these rays, traversing the prism at a larger angle than those from the center, will be refracted more towards the shorter wavelength portion of the spectrum. Pertinent to this is an important theorem verified by Uhler[5] stating that the minimum deviation of the actual oblique rays is less than the minimum deviation of their projections on a principal section which itself is greater than the minimum deviation of a ray of the same wavelength refracted in the principal section—the latter being an absolute minimum. These excellent papers by Uhler give a comprehensive discussion of the oblique refraction by a prism.

Preliminary to the derivation of prismatic curvature is the establishment of two theorems

[1] W. W. Coblentz, *Investigations of Infrared Spectra* (Carnegie Institution of Washington, D. C., 1905), p. 20.
[2] H. Kayser, *Handbuch der Spectroscopie* (S. Hirzel, Leipzig, Germany, 1900), Vol. I, p. 319.
[3] C. Schaefer and F. Matossi, *Das Ultrarote Spectrum* (Julius Springer, Berlin, Germany, 1930), p. 51.
[4] G. Subrahmaniam and D. Gunniaya, Phil. Mag. **48**, 896 (1924).
[5] H. S. Uhler, Am. J. Sci. **27**, 223 (1909); **35**, 389 (1913); J. Opt. Soc. Am. **6**, 1072 (1922).

regarding oblique optical refraction. Although Southall[6] derives these geometrically, they will be confirmed here by the use of Fermat's principle which states that *the path taken by a light ray in going from one point to another through any set of media is such as to render its optical path equal, in the first approximation to other paths closely adjacent to the actual one.* In Fig. 1 the plane T represents the boundary between two media of indices n and n'. The plane of the paper represents a normal section. The incident ray RO makes an angle η with the projection PO of RO on the normal section. In the normal section the projection makes an angle i with the normal to the surface. In the medium of index n' the angle that $R'O$ makes with the projection $P'O$ of $R'O$ on the normal section is η'. In the normal section the projection of the refracted ray makes an angle r with the normal to the surface.

Let the incident ray strike the boundary at a general point with coordinates $(x, 0, z)$. The optical path from R to R' in Fig. 1 may be written as

$$\begin{aligned} d &= n(RO)+n'(R'O) \\ &= n[(a+x)^2+b^2+(c+z)^2]^{\frac{1}{2}} \\ &\quad +n'[(a'-x)^2+(b')^2+(c'-z)^2]^{\frac{1}{2}}. \end{aligned} \quad (1)$$

Fermat's principle requires that the first derivative of d with respect to x and z must vanish in order that d be the optical path. Therefore

$$\frac{\partial d}{\partial x}=\frac{n(a+x)}{[(a+x)^2+b^2+(c+z)^2]^{\frac{1}{2}}} +\frac{n'(a'-x)}{[(a'-x)^2+(b')^2+(c'-z)^2]^{\frac{1}{2}}}=0, \quad (2)$$

$$\frac{\partial d}{\partial z}=\frac{n(c+z)}{[(a+x)^2+b^2+(c+z)^2]^{\frac{1}{2}}} +\frac{n'(c'-z)}{[(a'-x)^2+(b')^2+(c'-z)^2]^{\frac{1}{2}}}=0. \quad (3)$$

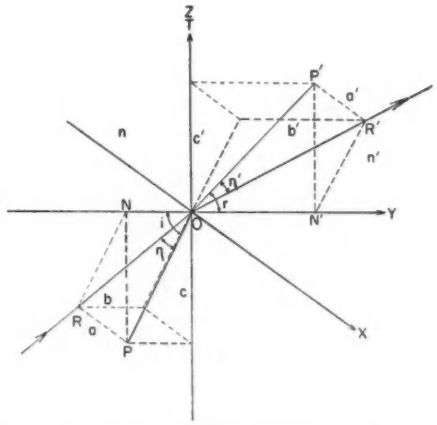


FIG. 1. Oblique refraction at a plane boundary between two media of indices n and n'.

Equation (2) results in

$$\frac{n(a+x)}{RO}=\frac{n'(a'-x)}{R'O},$$

which can be written as

$$n\sin\eta=n'\sin\eta'. \quad (4)$$

This law of refraction for an oblique ray states that the ratio of the sines of the angles which the incident and refracted rays make with the plane of any normal section of the refracting surface is a constant given by n'/n. Equation (3) results in

$$\frac{n(c+z)}{RO}=\frac{n'(c'+z)}{R'O}$$

or

$$\frac{n(c+z)}{PO/\cos\eta}=\frac{n'(c'+z)}{P'O/\cos\eta'},$$

which can be written as

$$n\cos\eta\sin i=n'\cos\eta'\sin r. \quad (5)$$

This second law of refraction applicable to an oblique ray states that: The projections of the incident and refracted rays on a plane of a normal section of the refracting surface are also subject to the law of refraction providing the indices of refraction are modified by the cosines of the angles η and η'. It is evident that if the optical path is in a normal section $\eta=\eta'=0$, and this relationship becomes the familiar Snell's law.

The passage of the rays from the entrance slit S_1S_2 through the prism to the curved image $S_1'S_2'$ is shown in Fig. 2 without the collimator and telescope interposed. The angles η_1, η_1', η_2', and η_2

[6] J. P. C. Southall, *Principles and Methods of Geometrical Optics* (Macmillan Company, New York, 1910), p. 28.

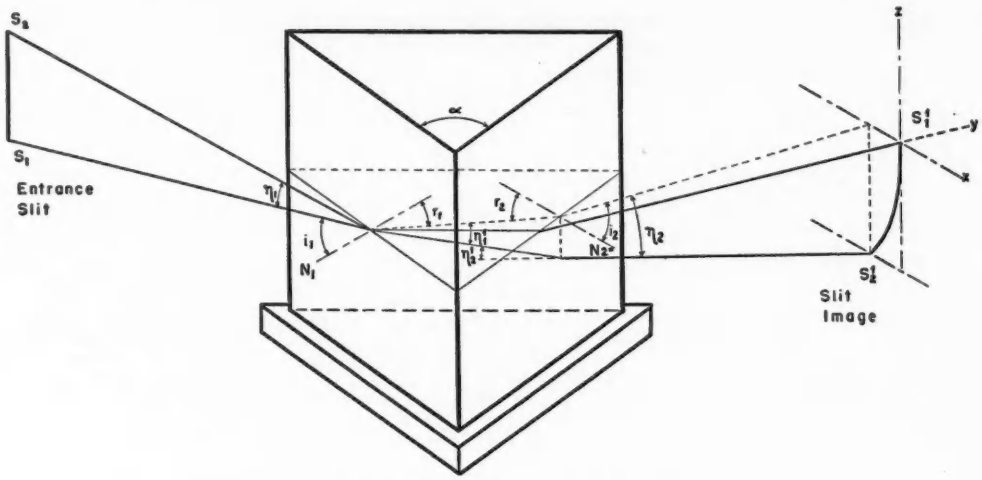


FIG. 2. Prismatic curvature of slit image.

between the oblique rays and the principal section are measured in a vertical plane parallel to the refracting edge. In Fig. 2, i_1, r_1, r_2, and i_2 are the angles that the projected rays in the principal plane make with the normals. The prism will have an index of refraction n relative to the surrounding media taken to be the same on all sides. Under this condition angles $\eta_1=\eta_2=\eta$. Since the principal section is the same normal section for both refracting faces, $\eta_1'=\eta_2'=\eta'$. The indices 1 and 2 will refer to the first and second prism face encountered, respectively. The primed letters apply within the prism. By applying Eqs. (4) and (5) at each face one obtains

$$\sin\eta = n\sin\eta' \tag{6}$$

$$\cos\eta\sin i_1 = n\cos\eta'\sin r_1 \tag{7}$$

$$\cos\eta\sin i_2 = n\cos\eta'\sin r_2 \tag{8}$$

and, in addition,

$$\alpha = r_1 + r_2, \tag{9}$$

where α is the refracting angle of the prism. The familiar expressions for refraction in a principal section are evident here for $\eta=0$.

For convenience define a new index of refraction in the following manner:

$$\nu = n\cos\eta'/\cos\eta. \tag{10}$$

Then Eqs. (7) and (8) assume the form of those for refraction in a principal section, with ν replacing n. From Eq. (6) one obtains

$$(\cos\eta')^2 = (1-\sin^2\eta') = (n^2-\sin^2\eta)/n^2. \tag{11}$$

Square Eq. (10) and substitute for $\cos^2\eta'$ from Eq. (11). Hence

$$\nu^2 = n^2\left(\frac{\cos\eta'}{\cos\eta}\right)^2 = \left[\frac{n^2-\sin^2\eta}{n^2}\right]\left[\frac{n^2}{\cos^2\eta}\right]$$

$$= 1+(n^2-1)\sec^2\eta. \tag{12}$$

Therefore

$$\nu = [n^2+(n^2-1)\tan^2\eta]^{\frac{1}{2}}$$

$$\cong n+\frac{n^2-1}{2n}\tan^2\eta \text{ (for small } \eta) \tag{13}$$

$$\Delta n = \nu - n \cong \frac{n^2-1}{2n}\tan^2\eta. \tag{14}$$

In order to determine the curvature of the slit image it will be convenient to obtain an analytical equation for the curved image with respect to a coordinate system based at the axial point of the image. As shown in Fig. 2 the z axis will be parallel to the slit and to the refracting edge of the prism, the y axis will be along the optic axis, and the x axis normal to these two. Let the focal lengths of the collimator and telescope be equal and denoted by f. Then

$$z = f\tan\eta \tag{15}$$

$$x = f\tan\Delta\theta, \tag{16}$$

where $\Delta\theta$ is the small angular lateral separation caused by the dispersion of the prism. The problem now is to relate x and z to the instrumental parameters n and f:

$$\Delta\theta=\frac{d\theta}{dn}\Delta n\cong\frac{d\theta}{dn}\frac{n^2-1}{2n}\tan^2\eta. \tag{17}$$

Substituting into Eq. (16) and approximating the tangent of the angle by the angle itself, we have

$$x=f\frac{n^2-1}{2n}\frac{d\theta}{dn}\tan^2\eta. \tag{18}$$

From Eq. (15) it is seen that $\tan\eta=z/f$. Upon substitution into Eq. (18) the equation of the curved image is seen to be a parabola,

$$x=\frac{n^2-1}{2nf}\frac{d\theta}{dn}z^2. \tag{19}$$

In order to determine the radius of curvature, one makes use of the well-known formula for the radius of curvature obtained from the calculus

$$\rho=\frac{[1+(dz/dx)^2]^{\frac{3}{2}}}{\dfrac{d^2z}{dx^2}}.$$

Differentiating results in

$$\rho=\frac{nf}{(n^2-1)}\frac{dn}{d\theta}\left[1+4\left(\frac{n^2-1}{2nf}\right)^2\left(\frac{d\theta}{dn}\right)^2z^2\right]^{\frac{1}{2}}. \tag{20}$$

Upon evaluating at the vertex we have

$$\rho=\frac{nf}{n^2-1}\frac{dn}{d\theta}. \tag{21}$$

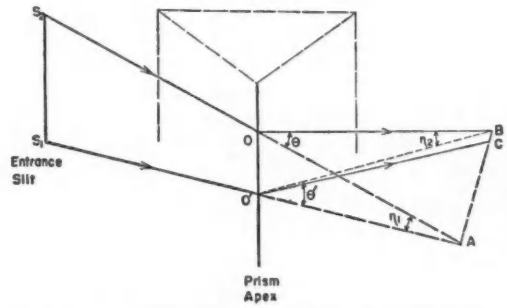


FIG. 3. Angular relationships between incident and refracted rays and their projections on a principal section when transposed to the prism apex.

For a prism at minimum deviation, when $i_1=i_2$ and $r_1=r_2$, the well-known relationship

$$n=\frac{\sin\frac{1}{2}(\alpha+\theta_{\min})}{\sin\frac{1}{2}\alpha} \tag{22}$$

results in

$$\frac{dn}{d\theta_{\min}}=\frac{(1-n^2\sin^2\frac{1}{2}\alpha)^{\frac{1}{2}}}{2\sin\frac{1}{2}\alpha}. \tag{23}$$

Although applying exactly only to those rays in a principal section, it may, nevertheless, be used as sufficiently correct for rays of slight obliquity. The radius of curvature at the vertex of the curved slit image for minimum deviation becomes

$$\rho\cong\frac{nf(1-n^2\sin^2\frac{1}{2}\alpha)^{\frac{1}{2}}}{2(n^2-1)\sin\frac{1}{2}\alpha}. \tag{24}$$

Equation (23) can be simplified to

$$\frac{d\theta}{dn_{\min}}=\frac{2}{n}\tan i,$$

where i is the angle of incidence of a ray refracted at minimum deviation in the principal section. Then the radius of curvature may be written as

$$\rho\cong n^2f\cot i/2(n^2-1). \tag{25}$$

It is easily seen that for the successive passage through m prisms, this becomes

$$\rho\cong n^2f\cot i/2m(n^2-1). \tag{26}$$

It is sometimes more desirable to present the curvature in terms of the sagittal distance x rather than by ρ. By substituting Eq. (21) into Eq. (19) there results

$$x=z^2/2\rho, \tag{27}$$

where z of this formula generally would be taken to be the distance of the slit extremity from the principal section multiplied by the magnification factor of the collimator-telescope system.

The curvature expressed in terms of the angular deviation difference between an oblique ray and one in the principal section may be written by substituting Eq. (21) into Eq. (17):

$$\Delta\theta=(f/\rho)\tan^2\eta. \tag{28}$$

An expression of similar meaning as derived by Subrahmaniam[1] and Gunniaya is arrived at in

the following manner. Figure 3 is designed to show the angular deviations between the incident and emergent rays in Fig. 2. Figure 3 is obtained from Fig. 2 by translating the oblique incident and emergent rays to the prism vertex forming S_2O and OB. Also their projections on the principal section are translated to the vertex forming S_1O' and $O'B$. The extensions of S_2O and S_1O' result in OA and $O'A$. In this figure θ is the actual deviation between the incident and the emergent oblique rays, while θ' is the angle of deviation between their projections in the principal section. The actual emergent ray in the principal section is also indicated as $O'C$ in order to show that its deviation is less than θ'. The angle $\eta_1=\eta_2=\eta$ is between the incident or refracted ray and their projections on the principal section. Since the surrounding medium is the same on both sides of the prism, $OA=OB$ and $O'A=O'B$. It is easily seen that $AB=2OA\ \sin\frac{1}{2}\theta$ and that $AB=2O'A\ \sin\frac{1}{2}\theta'=2OA\ \cos\eta\ \sin\frac{1}{2}\theta'$. Hence,

$$\sin\tfrac{1}{2}\theta=\sin\tfrac{1}{2}\theta'\cos\eta, \tag{29}$$

where $\cos\eta$ is a positive quantity less than unity; therefore $\theta'\geq 0$. If $2l$ is the length of the slit and f the distance of the collimating lens or mirror from the slit, then

$$\cos\eta=f/(f^2+l^2)^{\frac{1}{2}}. \tag{30}$$

The form given by Schaefer and Matossi[1] is therefore

$$\sin\tfrac{1}{2}\theta=\frac{f}{(f^2+l^2)^{\frac{1}{2}}}\sin\tfrac{1}{2}\theta'. \tag{31}$$

Presumably most investigators employing this form at or near to minimum deviation assume θ to be given by the usual expression, Eq. (22). Strictly speaking, this is incorrect by virtue of the fact that the ray in the principal section is not deviated as much as the projection of the oblique ray is. For small obliquity this approximation may be justified. Results of similar form could be obtained by use of Eq. (17).

Numerical values useful for a determination of the prismatic curvature at the vertex of a spectral line of parabolic form are given in Fig. 4. Here the ratio of the radius of curvature to the focal length of the telescope is plotted as ordinate with the index of refraction as abscissa resulting

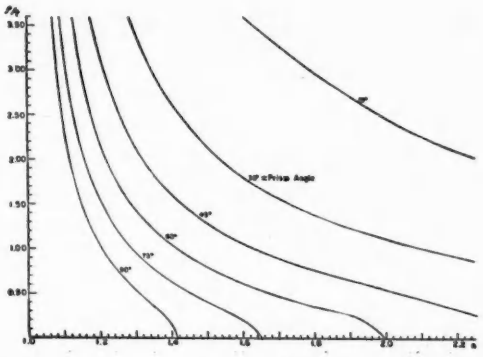


FIG. 4. Radius of curvature of slit image to focal length *versus* index of refraction.

in a family of curves at constant prismatic refracting angle. The curvature in terms of x or $\Delta\theta$ can be obtained by means of Eqs. (27) and (28), respectively. For a particular prism angle α there will be a maximum index of refraction beyond which critical reflection will take place at the second face. This occurs when $\sin\frac{1}{2}\alpha=1/n$.

The indices of refraction of CaF range from 1.43 at 0.8 microns to 1.32 at 9.4 microns and for NaCl from 1.54 at 0.6 microns to 1.44 at 15.9 microns. These substances are normally used as 60° prisms. Most modern infrared prism monochromators employ the Littrow-Wadsworth system, whereby the radiation passes through the prism twice. Therefore the values obtained from Fig. 4 must be divided by 2. Less common prismatic materials such as AgCl have refractive indices ranging from 2.1 at 0.5 microns to 1.9 at 20.0 microns, while TlBrI, referred to as KRS-5, has values from 2.63 at 0.58 microns to 2.22 at 40 microns. The KRS-5 prism generally has a refracting angle in the neighborhood of 30° or less. The curvature correction established for a 60° NaCl prism in the infrared will correspond approximately to that required for a 30° KRS-5 prism beyond about 20 microns.

Often rather than selecting an average value for the curvature, compensation is made where the resolving power is a minimum because of $dn/d\lambda$. For NaCl this is at 2.9 microns where the index of refraction is 1.5245.

The curvature of spectral lines also occurs upon reflection from a plane grating. Minkowski[7] has

[7] R. Minkowski, Astrophys. J. 96, 306 (1942).

derived the following relationships for this case:

$$x = \frac{\lambda z^2}{2f^2(d\lambda/dx)}$$

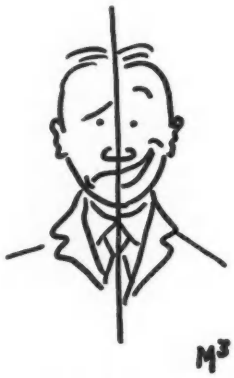
$$\rho = f^2 d\lambda/\lambda dx,$$

where λ is the wavelength of the radiation, and x, as in our case, is the distance from the center of the spectral line normal to the optic axis in the direction of the dispersion with z parallel to the entrance slit. In contrast to the prismatic distortion the curvature due to a grating is concave towards the longer wavelengths.

The author wishes to express his gratitude to Mr. Frank Speck for the drawing of Fig. 2.

Comic Dictionary of Physical Terms

Illustrated by
MARGE M. MULLER
East Lansing, Michigan



Phase Modulation

Planck's Law

Simple Harmonic Motion

Stark Effect

The Significance of Complementarity for the Life Sciences*

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(Received February 6, 1952)

A suggestion of Bohr's to the effect that numerous situations exist among the life sciences and the social sciences which exhibit characteristics of complementarity strikingly parallel to those of quantum physics is developed further in this paper. The supposition is advanced that there may exist for biological systems inherent relations of indeterminacy between sets of observable properties of such systems analogous to the Heisenberg indeterminacy relations between canonically conjugate pairs of observables in quantum mechanics. The existence of such indeterminacies would be manifested, as it is in physical systems, in disturbances, uncontrollable in principle, produced when such systems interact with instruments and apparatus used to gain information about them. The fact that radical disturbances, whether incidental or inherent, accompany the process of observation in the life sciences is, of course, quite obvious. If a well-defined portion of such disturbances should turn out to be inherent, indeterminacy and probability would be introduced into biological phenomena as basic elements. Further progress in scientific explanation and synthesis of knowledge in these fields would then require a redefinition of the state of a biological system analogous to the necessity for using quantum ψ-functions for specifying the state of a physical system with the attendant possibility of wholly new insights, such as those in physics which ultimately depend on the symmetry properties of ψ-functions. This new, and presently unknown, scheme for specifying the state of biological systems would then presumably result in the typically complementary aspects of phenomena in these fields to which Bohr has called attention. A number of familiar examples of apparently complementary phenomena among the biological and social sciences are described, and the possibility that they may arise from inherent indeterminacies basic to the systems which exhibit these phenomena is considered. It is concluded that these considerations lead to the expectation that an approach along such lines in future investigations in the life sciences holds considerable promise, and that the physics teacher is in a unique position to assist in stimulating such an approach and in pointing out by analogy the means at each stage which are likely to prove best suited for carrying it through.

ONE of the most characteristic features of the whole development of science has been the way in which successful techniques and working principles found in one field of investigation have stimulated ideas of possible approaches in other fields. Attempts to develop a scientific mode of explanation in any field of inquiry must start with the formulation of fruitful and productive hypotheses. The choice of such hypotheses is largely intuitive and often has been made on the basis of analogy with the kinds of assumptions which have proven fruitful in other fields of investigation. In this process physics, by virtue of the comparative simplicity of the systems which it treats, has frequently been the source of such hypotheses for other sciences. It is to this indirect role of physics in stimulating fruitful lines of investigation in other fields, as opposed to an account of progress made within its own proper confines, that the present paper is addressed. For the teacher of physics, as opposed to the investigator, this indirect role is always latent as a concern which commands his attention and his interest. It, therefore, seems appropriate and of some value and interest for us to turn our attention at this meeting of physics teachers to certain implications of modern physics which may well prove to be of significance in investigations in the life sciences.

Professor Niels Bohr at several stages in the development of his concept of complementarity in physics has pointed out the existence of situations in the life sciences which exhibit characteristics of complementarity as striking as any (e.g., the wave and corpuscular properties of the elementary particles) in quantum mechanics. His suggestions in this respect are concisely summarized in a recent paper by him in *Science*.[1] It will be helpful for the subsequent development of these ideas which will be attempted here to quote this portion of Bohr's paper in full:

> The epistemological lesson we have received from the new development in physical science, where the

* Given at the Annual Meeting of the American Association of Physics Teachers, Barnard College, New York City, February 1, 1952.

[1] N. Bohr, Science 111, 51 (1950).

problems enable a comparatively concise formulation of principles, may also suggest lines of approach in other domains of knowledge where the situation is of essentially less accessible character. An example is offered in biology, where mechanistic and vitalistic arguments are used in a typically complementary manner. In sociology, too, such dialectics may often be useful, particularly in problems confronting us in the study and comparison of human cultures, where we have to cope with the element of complacency inherent in every national culture and manifesting itself in prejudices which obviously cannot be appreciated from the standpoint of other nations.

Recognition of complementary relationship is not least required in psychology, where the conditions for analysis and synthesis of experience exhibit striking analogy with the situation in atomic physics. In fact, the use of words like *thoughts* and *sentiments*, equally indispensable to illustrate the diversity of psychical experience, pertain to mutually exclusive situations characterized by a different drawing of the line of separation between subject and object. In particular, the place left for the feeling of volition is afforded by the very circumstance that situations where we experience freedom of will are incompatible with psychological situations where causal analysis is reasonably attempted. In other words, when we use the phrase "I will" we renounce explanatory argumentation.

Altogether, the approach towards the problem of explanation that is embodied in the notion of complementarity suggests itself in our position as conscious beings and recalls forcefully the teaching of ancient thinkers that, in the search for a harmonious attitude toward life, it must never be forgotten that we ourselves are both actors and spectators in the drama of existence. To such an utterance applies, of course, as well as to most of the sentences in this article from the beginning to the end, the recognition that our task can only be to aim at communicating experiences and views to others by means of language, in which the practical use of every word stands in a complementary relation to attempts of its strict definition.[2]

It is just this suggested significance of the notion of complementarity, as providing possible new fruitful approaches for investigations in such fields as biology and psychology, that I propose to develop here. By way of doing so, it will be well first to review briefly the way in which complementary features of physical systems arise in quantum mechanics. These of course are all ultimately traceable to the limitations, as formulated in the Heisenberg principle of indeterminacy, on the precision with which values can be simultaneously specified for pairs of canonically conjugate variables. It is just this novel appearance of indeterminacies with their attendant statistical concomitants involving probabilities as basic attributes of physical systems that accounts for the unique features of quantum mechanics. An analysis of the origin of these indeterminacies in specific physical situations, such as, for example, in Heisenberg's original discussion of the gamma-ray microscope, reveals that the physical operation of determining the value of some physical property of a system involves exchanges of energy and momentum which are uncontrollable in principle and which result in a necessary latitude in the value of the canonically conjugate property of the system of just the amount specified by the indeterminacy relation.

This has become perhaps the most commonly offered explanation of the meaning and origin of the indeterminacy principle. It is not, however, by any means all-inclusive since many cases in which the indeterminacy principle is a controlling factor do not involve any experimental arrangement to which the system is subjected or any directly associated act of measurement or determination of the values of physical quantities. The familiar relations between the range of nuclear forces and the uncertainty in momentum attendant on the creation of a virtual π-meson, and the relation between the lifetime for a transition and the line breadth or uncertainty in the energy of the transition are two cases in point.

The incorporation of this principle into the subject of mechanics is achieved by means of the parallel commutation relations between pairs of variables related by the indeterminacy relation. Such variables then become operators which act on the ψ-functions or state functions which in quantum mechanics define the state of the system. Such functions contain all the information available about the state of a physical system and serve in quantum mechanics in place of specifications of coordinates and momenta employed in classical mechanics for the definition of state. Reflecting the inherent indeterminacies to which the use of a ψ-function to specify the state of a system owes its existence, the values of coordinates and momenta in a state so specified are characterized only by their probabilities of occurrence.

[2] Reference 1, p. 54.

One of the most striking features of quantum-mechanical systems whose states are described by ψ-functions is the sharp contrast in which the behavior of the system often stands when determinations of various physical properties are made on it. These contrasts are identified by Bohr as constituting his notion of complementarity. Thus, electromagnetic radiation exhibits the well-known contrasting wave and corpuscular phenomena involving on the one hand interference and diffraction effects and on the other the photoelectric effect, Compton scattering, and pair formation. The complementary features are often apparent simultaneously in the same experimental arrangement as, for example, in recording interference fringes by means of a photographic plate where the individual grains in the emulsion appear on absorption of the individual photons. Many ingenious arrangements have been suggested, notably by Einstein and his collaborators, to force a resolution of the complementary aspects of such phenomena or at least to demonstrate a logical inconsistency with the tenets of quantum mechanics in instances in which they appear. So far all such attempts have proved futile, and most physicists are resigned to the acceptance of complementarity as an unavoidable characteristic of our description of the physical world.

Before going on to my primary objective of inquiring into the possible significance of this radical experience in physical science for investigations in the life sciences, there is an important question which needs to be faced. This is essentially the question of the extent to which the use of such terms as "indeterminacy principle" and "complementarity" in this discussion involves a commitment to a particular point of view with respect to the nature of physical reality. As has already been noted, one point of view which has been fairly widely held sees the occurrence of indeterminacies and the associated complementary features of phenomena as arising from the operation of uncontrollable disturbances imposed on the system by the process of gaining information or knowledge about it through the use of appropriate experimental apparatus. In this view, the peculiar features of quantum mechanics are not so much representative of inherent characteristics of physical nature as they are of the necessary limitations imposed by our observation of it. Another view, to which I myself incline, has been developed by Margenau with great thoroughness in numerous papers on the subject and in his book, *The Nature of Physical Reality*.[3] In this view the indeterminacies and probabilities characteristic of the quantum-mechanical description of physical systems are real characteristics of the atomic world. Although they frequently become apparent as the system interacts with the apparatus employed in its investigation, they are there as real attributes of physical systems wholly apart from the inadvertencies introduced by apparatus. But, however this may be, no commitment to one or the other viewpoint is involved in the main course of the argument to be developed here. As we shall see, for our purpose the notion of complementarity can be taken in a purely descriptive sense as implying only that quantum systems do in fact exhibit contrasting phenomena, such as the wave and corpuscular behavior of radiation and material particles, which stand in a complementary relation to each other. The deeper epistemological questions which such a recognition immediately raises do not *have* to be resolved for the purpose in hand.

With this background with respect to the situation in physics, let us now turn to the possible significance which it may have for investigations in the life sciences. Here we shall propose the possibility that something akin to the indeterminacy principle in physics may exist for biological systems and as a result ultimately require a modification in the manner in which the state of such a system can be specified. If such a relation should indeed prove to be operative in biological systems, those properties of the system connected by indeterminacy relations would, as in the case of physical systems, lead to the contrasting phenomena involving them which Bohr designates as complementary. A variety of phenomena among the life sciences, which appear to stand in strikingly complementary relationships to each other, can be pointed to as evidence of the possible fruitfulness of such an approach in future investigations in these fields.

It is a well-recognized and a quite central

[3] H. Margenau, *The Nature of Physical Reality* (McGraw-Hill Book, Company, Inc., New York, 1950).

characteristic of the life sciences that the role played by the disturbance produced by the process of observation on the subject being observed is far more dominant and significant in them than it is in the physical sciences. Many of the significant observations in biology result in the destruction of the organism as a necessary consequence of the observational procedure. In this respect, they are analogous to observations on photons in physics which for the most part necessitate the absorption and disappearance of the particle being observed. Everything done to an individual under study by a psychologist has the effect of changing his state from that which would have occurred had the study not been made at all. The same situation holds in an even more acute form in the case of groups of individuals.

The precise character and role of the effect on an observation of the process of making it has not yet been formulated for the life sciences. It is conceivable that a careful analysis of this role would reveal the ultimate possibility of reducing the magnitude of such effects without limit, at least to a level at which they could be ignored. In that event such influences would constitute merely a problem in experimental technique, as they do in classical physics, and their action would be extraneous to the properties and behavior of the systems under investigation. It is undoubtedly in this light that they are currently regarded by the majority of biologists, psychologists, and other investigators in these fields. On the other hand, a careful analysis, under which they are reduced to idealized situations comparable to those employed in the examples used by Heisenberg and Bohr in physics, might well reveal the existence of a minimum level, independent of the particular details of special observational procedures, below which the disturbance produced in the system by the carrying out of any such procedure could not be reduced by any further ingenuity. The existence of such a level inherent in biological systems as such would be of the most profound importance for the future course of scientific investigation and description in these fields. It would presumably involve inherent indeterminacies characteristic of the system under investigation and not capable of being circumvented by experimental ingenuity. Such indeterminacies would lead to a behavior on the part of systems possessing them of a complementary nature characterized by sharply contrasting phenomena. But as already indicated in the passage quoted from Bohr's paper, a variety of situations among the life sciences do indeed appear to exhibit features of a distinctly complementary character. It is because of the existence of such features that the present suggestion is offered as an indication of the possible fruitfulness of the novel concepts developed in quantum mechanics for the analysis and synthesis of experience among the life sciences.

It will be helpful to consider a few examples of such complementary features among the life sciences. One of the most striking of these cases is to be found in the behavior of biological systems. On the one hand, it is now clear that biological systems are quite definitely physico-chemical mechanisms. Every living cell is a complex biochemical factory in which the most varied and intricate syntheses and degradations of compounds go on all the time. Radical differences in ionic concentrations are maintained across its surface. It continually takes in substances from the medium in which it lives; these it modifies and rebuilds in complex enzymatic processes, and the wastes it discharges across its boundary back into the medium. More and more biochemistry is revealing some of the details of these processes, and our knowledge of the structures of the compounds and the kinetics and mechanism of their metabolism grows steadily richer and more detailed. There can scarcely be any question any longer that living organisms are indeed physico-chemical mechanisms. To be sure, these mechanisms are of vast complexity and intricacy, but at the same time each component process or reaction seems to be understandable in terms of chemical structure, bond-energies, ionic solutions, electrokinetic phenomena, and the like.

But without in the least denying or refuting the physico-chemical mechanism aspect of living systems, they at the same time display a whole range of contrasting behavior so universal and essential in character as to be almost synonymous with the notion of "living system" itself. This is the aspect of biological organization by which an organism is characterized. Every plant, every

tree, every flower, every animal is an expression of this all-pervading organization in living things. A tree puts on a bit of growth each year all over. Billions of cells are involved in widely different places and yet they all grow in just the right way to preserve all the various angles and proportions which are proper to it. Some of the agents involved, such as hormones, are known, but how they are produced at just the right time, in the right amounts, distributed over the cells in just the right way to produce their action is not known at all. Embryology offers many fascinating instances of biological organization of the most amazing sort. Starting with a single germ cell, in the rapid successions of cell divisions a partly hollow spherical body is formed. Then come the many subsequent steps in which the continued growth of the hollow sphere develops at just the right moments differentiated cells with radically different structures and functions, each growing and developing at just the right rate in the right proportions at the right spot in the total aggregate. It is a steady march to a definite end with each event in step with all the rest. Moreover we can interrupt, damage, or attempt to alter the process in the greatest variety of ways and still come out with the same end result. Organisms have an amazing regenerative power and will bypass or circumvent in the most remarkable ways an artificial interruption or barrier placed by the biological investigator in the path of this steady goal-directed march.

A number of biologists have sensed the distinctly complementary character of these phenomena in living organisms and suggested the necessity of some radically new concept or approach if it is to be brought within the reach of experimental attack and assimilated in the framework of scientific description. After an extensive analysis of such contrasting phenomena Lillie concludes, "The general conclusion to which we are led by these considerations is that in living organisms physical integration and psychical integration represent two aspects, corresponding to two mutually complementary sets of factors, of one and the same fundamental biological process."[4] The wording of this statement is strikingly parallel to that which might be employed by a physicist with respect to the wave and corpuscular aspects of radiation. A recent book by Dean Sinnott gives a rich variety of examples along this same line. In it an expression is given to the need, to which the suggestion developed here is directed, for a fundamentally new approach to the subject. "Biology," he says, "has made enormous strides in the study of processes, of the successive series of chemical changes which go on in protoplasm; but these organizing relations which living things display present a much more formidable problem, and it may be that some new idea, some great generalization comparable to that of relativity for physics, will be necessary before we shall be able to understand the true nature of protoplasmic systems."[5]

Other examples of this same situation can be found in numerous other places among the life sciences. Much progress is being made currently in opening new avenues of understanding of the human nervous system through applications of information theory which draw upon, and employ, the insights gained in designing and operating the new high speed electronic digital computing machines. Psychologists, neuro-physiologists, and mathematicians are here joining hands in an intriguing new venture. Numerous striking parallels in the way in which each mechanism handles information have been uncovered. Operations such as programing, memory, neurosis, acceptance or rejection of alternate courses, and the like, all have their parallels in computing machines. Clearly this is a field of inquiry and investigation from which many new insights and understandings can be expected. At the same time, however, the type of behavior which the human nervous system so characteristically exhibits and which is represented by the words "responsibility," "sentiment," and "purpose" finds no counterpart in even the most elaborate electronic device which can be imagined. Indeed, such behavior stands in such sharp contrast to that represented by the words "thought," "response to stimulus," or "processing of information" as to seem to demand a complementary point of view for its description. Both are clearly

[4] Ralph S. Lillie, *The General Biology and Philosophy of Organism* (University of Chicago Press, Chicago, Ill., 1945), p. 50.

[5] Edmund W. Sinnott, *Cell and Psyche* (University of North Carolina Press, Chapel Hill, N. C., 1950), p. 41.

aspects of the behavior of one and the same physical system, and yet they represent a sort of mutual exclusiveness which seems to rule out the possibility of a unified explanation in common terms.

This same duality of explanation and expression characterizes the practical analysis and synthesis of experience beyond the present range of scientific explanation. Thus, psychosomatic methods in modern medical therapy stand in a typically complementary relationship to physiological and pharmacological methods. In the nonrationalized plane of everyday living on which all of us, from theoretical physicist to illiterate, must of necessity spend a considerable portion of our time, we do in practice unconsciously employ complementary modes of thinking about phenomena and that without particular concern with what we are doing. An illuminating example of this situation is provided by the contrasting points of view with which such problems of everyday living are presented to us by novelists. As a specific instance, imagine that we have before us two novels of equal excellence and depth of penetration. One presents its central character as caught up and carried along in the stream of life, his actions in each situation being made in response to social and environmental pressure and circumstance in accordance with the inescapable requirements of his heritage of limitations and prejudice. The other, with equal sensitivity and insight, presents its central character as a navigator piloting his way through the stream of life and reacting purposefully to the social and environmental pressures impinging on him in accordance with the essential requirements of the unity of his character and personality and the dictates of his will. In the one, human behavior is viewed as the resultant of the action of environmental stimuli on the whole complex of inherent characteristics which make up the whole human organism. In the other, such behavior is the expression of the inherent inner unity of the organism which, far from being determined by environmental stimuli, results in action on and modification of the environment. Provided each of these authors were equally able and sensitive artists, we would all doubtless accept both of their novels as equally true to life. Thus, even though we make no attempt to arrive at a rational understanding of the matter, we nevertheless agree in practice that the two aspects are equally valid. This then is clearly a case in which we employ in practice complementary modes of describing and appreciating a human life which are equally essential to a complete understanding but which, at the same time, stand in bold contrast to each other.

We find a similar situation in the interpretation we give to history in the practical business of carrying out our responsibilities as members of the community, the nation, and mankind as a whole. In this urgent business, we are all made acutely aware of an inherent indeterminacy in the situation which arises from the circumstance that we are at one and the same time both creatures and creators of history. From one point of view we feel ourselves entirely at the mercy of historic events and helplessly swept along with uncompromising inevitability in the current of our times. And yet at the same time, we cannot avoid the complementary conviction that our actions and our decisions are themselves the source of historic events and at every step direct and guide the current into new channels. From one point of view, we see America in these present disturbed times being relentlessly carried to the destiny which history has in store for her on the crest of Anne Lindbergh's "wave of the future." But at the same time, from another point of view, we see our country poised at a crucial turning point with the terrifying responsibility of the knowledge that the decision she makes now will shape that destiny for centuries to come.

This contrast is reminiscent of the ancient dictum referred to by Bohr in this connection that we must never forget that we are both actors and spectators in the drama of existence. As a purely practical matter we recognize here an inherent and basic indeterminacy as real and as operative as any in the field of physics. The question as to whether we are a creature or a creator of history is in fact reminiscent of the question which baffled prequantum physics as to whether light is a wave or particle phenomenon. In the light of our rich experience with the surprising answer given by quantum mechanics that it is in a real sense both at the same time, it is interesting to speculate as to whether the

answer to the former question may not prove ultimately to be of the same character.

The central problem with which we are dealing here really devolves around the question of the relative simplicity or complexity of the means which must be employed for an adequate scientific description of phenomena. This, however, involves profound philosophic problems which for scientists in general have been traditionally uninteresting if not actually distasteful. And yet there is no particular *a priori* reason why the experiential data of the life sciences should have to bear a simple direct correspondence with the conceptual synthesis of it which it is the goal of science to develop. Such a simple direct correspondence has had to be abandoned in order to advance at all in atomic and nuclear physics. In that field of investigation the state of a system is no longer an intuitively comfortable aggregate of observable or immediately perceivable physical properties but an abstract entity called a ψ-function. It could well be that the world is so constituted that biological systems too cannot be entirely understood or adequately treated by the causal synthesis of science until some such entirely new means of specifying the state of such a system is found. Indeed if the speculation advanced here should be borne out, and it should be found that the intricate and delicate systems of living organisms involve inherent indeterminacies which evidence themselves, as they do in the elementary systems of physics, in disturbances, uncontrollable in principle, produced when such systems interact with instruments and apparatus used to gain information about them, then it would become essential to discover such a redefinition of the state of these systems in order to proceed at all. Science, by definition, consists of causal synthesis of experience and so cannot deal directly with really indeterminate elements.

There is a further observation which can be made along this line of reasoning which has important implications for future investigation in these fields. In the case of physics the redefinition of the state of a physical system in terms of the ψ-functions of quantum mechanics has made it possible to understand and incorporate within the framework of scientific description phenomena which otherwise would have baffled comprehension. Notable among such instances are the great variety of phenomena which depend on symmetry properties of the ψ-functions. Such diverse and important subjects as homopolar bond energies in molecules, filled and conducting electronic bands in solids, alternating intensities in band spectra, and indeed, the exclusion principle itself, as well as the variety of phenomena which depend on considerations of parity would all be quite inexplicable if we were not able to describe states by ψ-functions having symmetry properties. Thus, the redefinition of state in quantum mechanics is much more than a device for dealing with a system which exhibits indeterminacy, probability, and complementary phenomena. This it accomplishes to be sure, but in addition it opens up new possibilities for understanding aspects of nature which would prove wholly foreign to the simpler conceptual pattern of classical physics. It is intriguing to speculate on the wholly new insights and understandings in the life sciences which might follow on such a radical redefinition of the state of a biological system, if the speculation advanced here should turn out to have some basis.

This then is the substance of the admittedly wholly speculative proposal which it has been my intention to develop. Although throughout this discussion nothing has been proved or demonstrated, still it is hoped that there may be suggestions here which physics teachers are in a unique position to provide for oncoming investigators in other fields, and which on an off-chance might somewhere stimulate a fresh insight which could open up new and immensely fruitful avenues for the synthesis of experience in areas far removed from physics. All of us are, in the end, partners in the fascinating quest for comprehension of the strange and intricate world which we inhabit. We in modern physics have been forced to acknowledge that reality in that world is not as simply and directly related to intuitive experience as our predecessors felt it must be from their experience with classical physics. But our abandonment of what Margenau calls simple "rules of correspondence" between the constructs of physical theory and the data of immediate observation has proved a blessing in disguise and has opened for us wholly new ranges of phenomena which could be in-

corporated in the framework of our understanding and comprehension of the physical world. Several suggestive, if not convincing, arguments have been advanced which point to the possible existence of a very similar situation lying in wait for the life sciences, which so far have gotten along very well with only simple and intuitive rules of correspondence.

It is hoped that the manner in which these proposals have been advanced makes it clear that they are intended to suggest constructive new avenues for strictly scientific investigations in the life sciences, not a block or barrier to further scientific activity. This may, however, not be entirely clear from the context, particularly in view of the connotations and associations which some of the examples used generally carry with them. Indeed, Bohr has already given expression to this source of misunderstanding as may be seen from the following passage:

> Utterances of this kind would naturally in many minds evoke the impression of an underlying mysticism foreign to the spirit of science; at the above mentioned Congress in 1936 I therefore tried to clear up such misunderstandings and to explain that the only question was an endeavour to clarify the conditions, in each field of knowledge, for the analysis and synthesis of experience. Yet, I am afraid that I had in this respect only little success in convincing my listeners, for whom the dissent among the physicists themselves was naturally a cause of scepticism as to the necessity of going so far in renouncing customary demands as regards the explanation of natural phenomena.[6]

All of us do, in our nonscientific moments, acknowledge the validity of elements in our experience apart from or beyond our scientific approach to the comprehension of that experience. Many of the considerations which have been advanced here undoubtedly touch closely on such elements. Indeed, in my own case, I am sure that I give in my own thinking considerably greater weight and a higher validity to such elements than do the majority of my fellow workers in science. However, I do hope it will be clear to you from the manner in which these suggestions have been put forward that nonscientific elements do not enter into the present proposal. If there should be some remaining doubt on this score, it will be fully resolved when it is considered that the entire proposal has been developed in close step-by-step analogy with quantum mechanics, itself an entirely scientific discipline. Any elements in the proposal which seem to be contrary to the spirit of science will, therefore, be found to have a parallel in modern atomic physics which will reveal that such elements have in fact a richly rewarding potentiality for increased comprehension and understanding of an entirely scientific character.

[6] N. Bohr, Chapter in *Albert Einstein: Philosopher-Scientist*, P. A. Schilpp, editor (Library of Living Philosophers, New York, 1951), p. 236.

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. 20, 252 (1952)].

Caswell, Randall Smith, Department of Physics, University of Kentucky, Lexington, Ky.

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Koranda, Edward, Rt. 3, Lincolnia Park, Alexandria, Va.

Mullin, Charles Raymond, (*J*), 7 College St., Canton, N. Y.

Peterson, Edward Charles, 5401 Glenwood St., Duluth, Minn.

Roberts, Francis Herbert, Dallas Theological Seminary, 3909 Swiss Ave., Dallas 4, Texas

Schwartz, Daniel S. (*J*), 255 Townsend St., New Brunswick, N. J.

Schwartz, Jack (*J*), 17 Seaman Ave., New York 34, N. Y.

Secunda, William Joseph (*J*), Box 3, O'Fallon, Ill.

Shockley, James Thomas (*J*), 1104 Franklin St., Apt. 1, Fresno, Calif.

Stajdohar, Ralph Edward (*J*), 1736 S. Ruble St., Chicago 16, Ill.

Streicher, Mitchell (*J*), 2718 Morris Ave., New York, N. Y.

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Measurement of the Specific Heat at Room Temperature of Copper in the Form of Wire by Electrical Heating

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(Received November 20, 1951)

The experiment described has been devised for the purpose of stimulating the interest of advanced students in methods already employed in accredited research; it also offers an exercise in the correction for heat losses by radiation and convection and indicates a method of minimizing conduction loss.

In the original work, the object was to determine the specific heat of nickel at temperatures up to and beyond the Curie temperature; for the purpose of simplification, however, the method has been applied to copper at the temperature of the laboratory.

MUCH can be done to lead students to appreciate successful research by the adaptation of methods and apparatus already used in such work; experiments so devised encourage reference to original papers and help to develop the faculty of criticism.

The particular experiment described here offers an excellent exercise in the application of a cooling correction; it has been adapted from experiments by E. Lapp[1] and K. E. Grew[2] in their determinations of the specific heat of nickel at various temperatures up to and beyond the Curie temperature of 350°C; in their work the specimen was heated in vacuum in a furnace to a known steady temperature which was measured by a thermocouple; a known and steady current was then passed through the specimen and its rate of heating measured; a correction was applied for heat losses by radiation, etc.

Lapp used a special wattmeter to measure the power input and Grew measured the current flowing through and the resistance of the wire, using for the latter purpose the same potentiometer as was used to measure the thermocouple emf. Then, if s is the specific heat of the specimen, m its mass, r its resistance and i the current it carries, and if t, θ, and J represent, respectively, time, temperature and electrical equivalent, then

$$s=i^2rdt/mJd\theta.$$

The main criticism of the method would seem to be that suggested by C. Sykes and H. Wilkinson,[3] namely that for materials in the form of wires the ratio of surface area to mass is very high, thereby necessitating large corrections due to radiation, convection, etc., particularly at high temperatures; these corrections require very accurate control of the pressure of the gas surrounding the specimen during the experiment, since fluctuation in the pressure above 5×10^{-3} mm of mercury will affect the correction to an appreciable extent.

It is important to note the consequences of adapting the method for use with copper, as has been done in this experiment. Let R, L, and D be the radius, length, and density of the wire, respectively. Then, if p is its resistivity and r its resistance, and the subscripts 1 and 2 refer to copper and nickel,

$$r=p_1L/\pi R_1^2=p_2L/\pi R_2^2,$$

provided the wires have the same resistance per unit length. Hence $R_1/R_2=0.4$ approximately, since $p_1/p_2\approx1/6.6$. Then, since area/mass $=2\pi RL/\pi R^2LD=2/RD=k$, we have $k_1/k_2=R_2D_2/R_1D_1\approx1/0.4=2.5$, since the densities of the two metals are nearly the same. Owing to the low resistivity of copper, its ratio of surface area to mass is, therefore, 2.5 times as great as for nickel for wires of the same resistance per unit length; thus, greater losses by radiation are likely to take place when using copper.

Also, because of the high thermal conductivity of copper compared with nickel, loss of heat by conduction may be expected to be more impor-

[1] E. Lapp, Ann. Physik **12**, 442 (1929).
[2] K. E. Grew, Proc. Roy. Soc. (London) **A145**, 509 (1934).
[3] C. Sykes and H. Wilkinson, Proc. Phys. Soc. (London) **50**, 834 (1938).

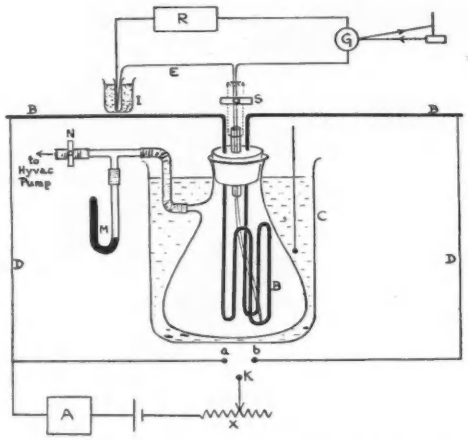

FIG. 1. Layout of apparatus showing the heating and thermocouple circuits.

tant for copper than for nickel; a method of minimizing this was found and is described later.

The experiment was carried out with the wire surrounded by air at low pressure.

APPARATUS

The arrangement of apparatus was as given in Fig. 1. About 60 cm of the central portion of a 120-cm length of bare No. 18 tinned copper wire B was bent into the shape shown and enclosed in a 250-cc conical vacuum filter flask connected to a Hyvac pump; the ends of the wire, each 30 cm long, were threaded through a rubber stopper and extended into the air outside and were there connected to the No. 24 double cotton-covered wire D of the heating circuit.

The heating current was by-passed by the switch K and allowed to attain a steady value, measured on ammeter A, before being passed through the copper wire in the flask; the current was controlled by a 7-ohm, 6.5-amp rheostat X and generated by a 6-volt battery.

The thermocouple was made from No. 32 enamelled copper and eureka (E) wires and was soldered to the copper specimen, using the minimum amount of solder which was reduced by scraping after the joint had been made; one junction, enclosed in a glass tube, was kept in melting ice I and a series resistance R of 5000 ohms was needed with the moving coil galvanometer G when using a lamp and scale.

The low pressure inside the conical flask was indicated by a small mercury manometer M, and the temperature of the flask was stabilized by immersing it in a 900-cc beaker of water C.

The thermocouple leads were taken out through the rubber stopper by using a short piece of glass tubing ending in rubber tubing closed by a screw clip S; this clip was used to open the flask to the atmosphere when required.

TABLE I. Heating and cooling of the copper wire as a function of time.

Heating		Cooling	
Time (min)	Thermocouple deflection (cm)	Time (min)	Thermocouple deflection (cm)
0.00	11.20	2.00	14.15
0.25	11.65	2.25	14.00
0.50	12.15	2.50	13.62
0.75	12.65	2.75	13.30
1.00	13.05	3.00	13.00
1.25	13.40	3.25	12.78
1.50	13.65	3.50	12.60
1.75	13.90	3.75	12.42
2.00	14.15	4.00	12.28
		4.50	12.05
Current off		5.00	11.90
		5.50	11.80
		6.00	11.70
		7.00	11.61
		8.00	11.58
		9.00	11.54
		10.00	11.51

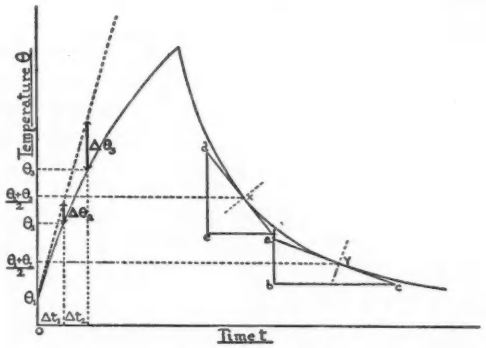

FIG. 2. Diagram of a heating and cooling curve to illustrate the method of calculation and application of the cooling corrections.

PROCEDURE

After obtaining a steady current with the connection $K-a$ closed (Fig. 1), the switch was changed to close $K-b$ so that the heating current began to flow through the copper wire in the

flask. Readings of the thermocouple deflection were taken every quarter minute for two or three minutes; the current was then switched off but the thermocouple deflection was recorded for another seven or eight minutes.

The thermocouple was calibrated by removing from the flask the copper specimens to which the hot junction was soldered and immersing them in water at a known temperature measured by a mercury thermometer; for such small temperature differences a linear relation between deflection and temperature was assumed.

Any serious leakage in the apparatus could be detected by closing the screw clips S and N after evacuation and noting the movement of the mercury levels in the manometer M; the pressure used in the experiment was such that little or no difference could be seen in the levels of the mercury in the two limbs of the manometer M; the pump was kept running throughout; it is interesting to note that when the pump was switched on the sudden expansion of the air in the flask caused considerable cooling of the wire, and some time had to be allowed to pass to enable it to regain its original temperature before commencing the heating.

TABLE II. Step-by-step temperature corrections deduced from Fig. 3.

Temp. θ (cm)	Mean temp. $\frac{\theta_1+\theta_2}{2}$ (cm)	Δt (min)	$d\theta/dt$ at mean temp. (cm/min)	Increment of correction $\Delta t \times d\theta/dt$ (cm)	Total correction $\Delta\theta$ at temp. θ (cm)
θ_1 11.20				0.00	0.00
	11.30	0.11	0.0		
θ_2 11.40				0.00	0.00
	11.50	0.11	0.0		
θ_3 11.60				0.00	0.00
	11.70	0.11	0.13		
θ_4 11.80				0.01	0.01
	11.90	0.11	0.27		
θ_5 12.00				0.03	0.04
	12.10	0.10	0.44		
θ_6 12.20				0.04	0.08
	12.30	0.09	0.50		
θ_7 12.40				0.05	0.13
	12.50	0.14	0.69		
θ_8 12.60				0.10	0.23
	12.70	0.13	0.80		
θ_9 12.80				0.10	0.33
	12.90	0.13	0.94		
θ_{10} 13.00				0.12	0.45
	13.10	0.14	1.09		
θ_{11} 13.20				0.15	0.60
	13.30	0.16	1.13		
θ_{12} 13.40				0.18	0.78

TYPICAL SET OF READINGS

(1) Calibration of Thermocouple

With cold junction at 0°C and hot junction at 16.4°C, and with the scale at 100 cm the deflection

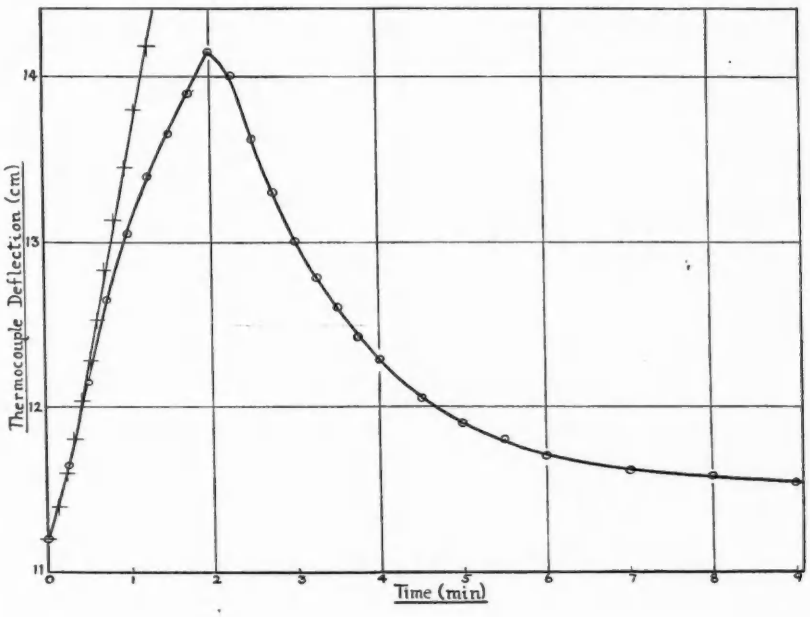

FIG. 3. Graph showing an actual heating and cooling curve plotted from the readings given in the text, together with the corrected heating curve.

was 11.0 cm. Thus 1-cm deflection represents 1.49°C.

(2) Other Readings

Diameter of copper wire = 0.122 cm (No. 18)
Density of copper wire = 8.93 g cm^{-3}
Resistivity of copper wire = 1.78×10^{-6} ohm-cm at 18°C
Current = 4.0 amp
Room temperature = 16.4°C.

Primary galvanometer readings are recorded in Table I.

The values of the diameter, density, and resistivity given above were taken from tables and are not necessarily the exact values for the No. 18 tinned copper wire used in the experiment.

CALCULATION OF CORRECTIONS FOR HEAT LOSSES BY RADIATION

The method employed was that in common use and is shown diagrammatically in Fig. 2. The correction at temperature θ_2 is $\Delta\theta_2$, where $\Delta\theta_2 = (ab/bc)\Delta t_1$. The correction at θ_3 is $\Delta\theta_3$, where $\Delta\theta_3 = \Delta\theta_2 + (de/ef)\Delta t_2$, etc.

The main source of error is that introduced in drawing the tangents to the cooling curve; to minimize this error a plane mirror was used to find the normals at X, Y, etc., and the tangents were then drawn perpendicular to the appropriate normals. The corrections are collected in Table II.

CALCULATION AND RESULT

Since

$$s = i^2 r dt/mJd\theta,$$

then

$$s = i^2 p dt/\pi^2 R^4 DJd\theta,$$

and using the values recorded in the readings for the constants in the above equation,

$$s = 0.000355 i^2 dt/d\theta.$$

From Fig. 3, the corrected rate of heating is 2.55 cm/min, which is equivalent to 3.80°C per minute. Hence, for a heating current of 4.0 amp,

$$s = 0.000355\times16.0\times60/3.80$$
$$= 0.090 \text{ cal/g deg C.}$$

GENERAL COMMENTS

The apparatus has been in general use by students whose results have consistently been within one percent of the above value. If closer agreement with the true value for pure copper is required, untinned wire should be used and particular care taken in the measurement of its diameter and in the calibration of the ammeter, since the specific heat varies inversely as the fourth power of the radius and directly as the square of the heating current.

When a short length of copper wire was used in the flask and the connection to the outside circuit was made using heavy brass connectors placed just outside the rubber stopper, the rise in temperature was small and the cooling rate large; application of the cooling correction previously described resulted in a corrected heating graph which was not linear; this was taken to indicate that the conduction loss was large. By making the copper-wire specimen of considerable length and also by extending it well beyond the rubber stopper, it was considered that the conduction loss might be minimized; this proved to be the case as the corrected heating graph obtained was linear, as shown in Fig. 3.

A university has its responsibility to the student as well as an obligation to add to knowledge. Let us not put our students in the position of observing or being a party to practices contrary to the ideal for a university and for his future career. If the student sees his institution engaged in fundamental research, freely supported and freely conducted, he will learn that independence in investigation is man's right and a university's responsibility to exemplify.—N. PAUL HUDSON, Ohio State University Graduate School Record, December, 1951.

The Scattering Theory of the Schrödinger Equation

DAVID PARK
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(Received November 12, 1951)

The Schrödinger equation for the scattered particle is converted into an integral equation, in which the kernel is obtained by the use of the condition that the events described by the solutions of the equation must be causally related, i.e., that causes must always precede the effects which they produce. The integral equation is iterated once, and it is found that its solution corresponds to the usual description of a scattering process in terms of an undeflected wave traversing the system plus a radially scattered wave moving outwards. In the second part, it is shown that a simple relation exists between solutions of Schrödinger's equation of the sort just described and the solution of the initial-value problem associated with the equation of motion for a free particle. By this means a solution of the scattering problem is set up which describes how the wave function develops as the wave which it describes propagates through the system. The series which results was first given by Feynman. As an example, the first Born approximation for the scattering problem is developed by each approach.

THIS note is prompted by the desire to give a treatment of the theory of scattering which is shorter than that usually found in the textbooks, and which at the same time parallels in its argument the recent developments in the theory of the S-matrix, through which more general problems are also being treated. It seems that the physical argument which lies behind the new mathematical methods has not explicitly been put in the simple way which follows, although the considerations which I shall present will surely be known to workers in the field.

SCATTERING AS A BOUNDARY-VALUE PROBLEM

Suppose we are to integrate the time-dependent Schrödinger equation,

$$[i\hbar\partial_t+(\hbar^2/2m)\Delta-V(\mathbf{r},t)]\psi(\mathbf{r},t)=0. \quad (1)$$

It is a familiar fact that if we can find functions $G(\mathbf{r},t)$ such that

$$(i\hbar\partial_t+(\hbar^2/2m)\Delta)G(\mathbf{r},t)=\delta(\mathbf{r})\delta(t), \quad (2)$$

then the solutions of Eq. (1) obey the equation

$$\psi(\mathbf{r},t)=\psi_0(\mathbf{r},t)$$
$$+\int G(\mathbf{r}-\mathbf{r}',t-t')V(\mathbf{r}',t')\psi(\mathbf{r}',t')d\mathbf{r}'dt', \quad (3)$$

where ψ_0 is any solution of the equation of motion for a free particle. Such a function is evidently given[1] by

$$G(\mathbf{r},t)=(2\pi)^{-4}\hbar^{-1}\int_C e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}/(\omega-\omega_k)d\mathbf{k}d\omega, \quad (4)$$

where $\omega_k=\hbar k^2/(2m)$ and the index C means that we have to specify the contour of the integrations around the singularity of the integrand. To determine C, it suffices to introduce a boundary condition which is not implicit in the differential equation (1), the postulate that the solutions of Eq. (1) at different times shall be causally related. More specifically, we require that $\psi(\mathbf{r},t)$ shall be determined entirely by the values taken by V and ψ at times previous to t, and shall be independent of what is to happen to them at later times. This is equivalent to saying that $G(\mathbf{r},t)$ is to vanish when t is less than zero.

We obtain the desired specialization of Eq. (4) if we integrate first over ω from minus infinity to plus infinity, choosing the contour C so as to pass above the singularity on the real axis at $\omega=\omega_k$. For if t is less than zero, we can close the contour in the upper half-plane, and since it now encloses no pole of the integrand, the integral vanishes. On the other hand, if t is greater than zero we close the contour below; now it passes clockwise around the pole and the integral over ω is just $-2\pi i\exp(-i\omega_k t)$. In this way we find that a solution of Eq. (2) which expresses causal re-

[1] The way in which functions of this kind are found is most easily seen if one applies to Eq. (4) the differential operator of the left side of Eq. (2).

lationships is

$$G(\mathbf{r},t)=\begin{cases}0 & (t<0)\\ -i(2\pi)^{-3}\hbar^{-1}\int e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_k t)}d\mathbf{k} & (t>0).\end{cases} \quad (5)$$

Let us now specialize the problem to be one in which V is independent of time and there is only one energy state occupied. In this case we can set

$$\psi(\mathbf{r},t)=\psi(\mathbf{r})e^{-i\omega_p t},$$
$$\psi_0(\mathbf{r},t)=\psi_0(\mathbf{r})e^{-i\omega_p t},$$

where $\hbar\omega_p$ is the constant energy of the system, and Eqs. (3) and (5) become

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})-i(2\pi)^{-3}\hbar^{-1}\int d\mathbf{k}d\mathbf{r}'\,V(\mathbf{r}')\psi(\mathbf{r}')e^{i\mathbf{k}\cdot\mathbf{r}-\mathbf{r}'}\times\int_{-\infty}^{t}e^{i(\omega_k-\omega_p)(t'-t)}dt'.$$

The last integral is of the form

$$\int_{-\infty}^{0}e^{i\omega t}dt=\lim_{\epsilon\to 0}-i/(\omega-i\epsilon). \quad (6)$$

It is related to the δ-function, and in the usual notation it is $2\pi\delta_+(\omega_k-\omega_p)$. With Eq. (6) we can write

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})-(2\pi)^{-3}(2m/\hbar^2)\lim_{\epsilon\to 0}\times\int\frac{d\mathbf{k}}{k^2-(p^2+i\epsilon)}\times\int e^{i\mathbf{k}\cdot\mathbf{r}-\mathbf{r}'}V(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}'. \quad (7)$$

In this, the angular k-integrals can be done at once, and the radial integral runs from zero to infinity, passing below the pole of the integrand at $(p^2+i\epsilon)^{\frac{1}{2}}$. We can express this situation in a more convenient way by deflecting the path of the k-integral to pass below the real axis at this point and then setting ϵ equal to zero. Denoting this contour by C_1, we can write Eq. (7) as

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})-\frac{1}{(2\pi)^3}\frac{2m}{\hbar^2}\int_{C_1}\frac{d\mathbf{k}}{k^2-p^2}\times\int e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}V(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}', \quad (7')$$

or, after the angular k-integral is done,

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})-\frac{2}{(2\pi)^2}\frac{2m}{\hbar^2}\int_{C_1}\frac{k^2dk}{k^2-p^2}\times\int\frac{\sin k|\mathbf{r}-\mathbf{r}'|}{k|\mathbf{r}-\mathbf{r}'|}V(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}'. \quad (7'')$$

It should be noted that an equation of this form is obtained at once if we start with the time-independent Schrödinger equation instead of with Eq. (1), and construct[1] the Green's function analogous to Eq. (4) with which to solve it. The only difference is that if we do this, we have no physical principle to guide us in the choice of a contour of integration. Even so, however, the problem can easily be done, for out of the set of solutions of the form Eq. (7') corresponding to different contours, we can select the one which solves the scattering problem we have in mind, i.e., in which the radially propagated wave (cf. Eq. (10)) has all its components directed outward. The required contour is of course C_1, as will be evident from the evaluation of Eq. (7'').

To complete the formal solution of the scattering problem we must evaluate the k-integral, and then find the asymptotic form of the resulting expression for ψ. The k-integral in Eq. (7'') is most easily done by means of the identity

$$\int_{C_1}\frac{kdk}{k^2-p^2}\sin k|\mathbf{r}-\mathbf{r}'|=-\frac{i}{2}\int_{C_2}\frac{kdk}{k^2-p^2}e^{ik|\mathbf{r}-\mathbf{r}'|}, \quad (8)$$

where C_2 runs from minus infinity to plus infinity, passing above the pole at $-p$ and below that at $+p$. (This is obtained by writing the sine as the sum of two exponentials and then changing the sign of k in the second.) Since $|\mathbf{r}-\mathbf{r}'|$ is necessarily positive, the path of integration can be closed by a semicircle in the upper half-plane. When this is done, only the pole at $+p$ falls within the contour, and the value of Eq. (8) may at once be written down as $\frac{1}{2}\pi\exp(ip|\mathbf{r}-\mathbf{r}'|)$. The resulting value of ψ is

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})-(4\pi)^{-1}(2m/\hbar^2)\times\int V(\mathbf{r}')\psi(\mathbf{r}')e^{ip|\mathbf{r}-\mathbf{r}'|}/|\mathbf{r}-\mathbf{r}'|d\mathbf{r}'. \quad (9)$$

If $V(\mathbf{r})$ decreases faster than $1/r$, which is generally true in practice, we can say that the contribution to the r'-integral in Eq. (9) from large values of r' is negligible, so that we can write

$$|\mathbf{r}-\mathbf{r}'|\sim r-\mathbf{n}\cdot\mathbf{r}',$$

where $\mathbf{n}$ is a unit vector parallel to $\mathbf{r}$. With $\mathbf{n}p=\mathbf{p}'$, we get

$$\psi(\mathbf{r})\sim\psi_0(\mathbf{r})-(4\pi)^{-1}(2m/\hbar^2)(e^{ipr}/r)\times\int V(\mathbf{r}')\psi(\mathbf{r}')e^{-i\mathbf{p}'\cdot\mathbf{r}'}d\mathbf{r}'. \quad (10)$$

This formula, by its derivation, is exact in its limit (the *wave zone* of the disturbance around the scattering center). It represents an undisturbed wave moving past the scattering center and a radially propagated wave which has its origin there. If Eq. (10) is solved by iteration, the first terms of the resulting series are

$$\psi(\mathbf{r})\sim\psi_0(\mathbf{r})-(4\pi)^{-1}(2m/\hbar^2)(e^{ipr}/r)\times\int V(\mathbf{r}')\psi_0(\mathbf{r}')e^{-i\mathbf{p}'\cdot\mathbf{r}'}d\mathbf{r}'; \quad (11)$$

this is the first Born approximation to the solution of the problem.

If $V(\mathbf{r})$ is spherically symmetrical, which we shall henceforth assume to be true, it is convenient to decompose ψ into partial waves; i.e., in quantum-mechanical terms, choose a representation in which the total angular momentum l is diagonal. We shall henceforth consider ψ_0 to be a plane wave. The wave function ψ becomes

$$\psi=e^{i\mathbf{p}\cdot\mathbf{r}}+\sum_{l=0}^{\infty}(2l+1)i^l g_l(r)P_l(\cos\theta), \quad (12)$$

where θ is the angle between $\mathbf{p}$ and $\mathbf{r}$, and $g_l(r)$ is a radial wave function[2] which is to be determined. Introducing the well-known expansion[3]

$$e^{i\mathbf{p}\cdot\mathbf{r}}=\sum_{l=0}^{\infty}(2l+1)i^l f_l(r)P_l(\cos\theta), \quad (13)$$

where

$$f_l(r)=(\pi/2pr)^{\frac{1}{2}}J_{l+\frac{1}{2}}(pr), \quad (14)$$

we can write ψ as

$$\psi=\sum(2l+1)i^l[f_l(r)+g_l(r)]P_l(\cos\theta). \quad (15)$$

[2] The factor of $i^l(2l+1)$ is introduced for convenience later (cf. Eq. (13)).

[3] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1933), p. 22.

Substituting this into Eq. (10) and doing the angular integrations,[4] we get

$$\sum(2l+1)i^l g_l(r)P_l(\cos\theta)\sim-(2m/\hbar^2)(e^{ipr}/r)\times\sum(2l+1)P_l(\cos\theta)\times\int_0^{\infty}[f_l(r)+g_l(r)]V(r)f_l(r)r^2dr,$$

and equating the coefficients of $P_l(\cos\theta)$ gives us the asymptotic formula

$$i^l g_l(r)\sim-i(e^{ipr}/2pr)T_l, \quad (16)$$

where

$$iT_l/(2p)=(2m/\hbar^2)\times\int_0^{\infty}[f_l(r)+g_l(r)]V(r)f_l(r)r^2dr. \quad (17)$$

With this result, we can use Eq. (15) to write down the asymptotic form of ψ, if we use the fact that for large values of pr,

$$f_l(r)\sim\sin(pr-\tfrac{1}{2}l\pi)/pr. \quad (18)$$

Substituting this and Eq. (16) into Eq. (15) and separating the exponentials gives

$$\psi(\mathbf{r})\sim(i/2pr)\sum(-)^l(2l+1)\times[e^{-ipr}-(-)^l(1+T_l)e^{ipr}]P_l(\cos\theta). \quad (19)$$

Evidently, the first term represents the totality of incoming waves and the second that of outgoing waves.

We can now establish the fact that the quantity $1+T_l$ is significantly restricted by the character of the problem which we have set ourselves. This problem brings with it two conservation laws: the number of particles leaving the scatterer is equal to the number arriving there, and the angular momentum of each particle, on account of the symmetry of V, is unchanged by the collision. The net number of particles leaving the system is given by the radial component of the asymptotic current vector formed with Eq. (19), integrated over all directions. The evaluation of this is straightforward, and gives

$$\int j_r d\Omega\sim-(\pi/mpr^2)\sum(2l+1)[1-|1+T_l|^2].$$

[4] The integration involves using the addition theorem for spherical harmonics and the orthogonality relation. The resulting formula is that given, for example, in W. Magnus and F. Oberhettinger, *Special Functions of Mathematical Physics* (Chelsea Publishing Company, New York, 1949), middle of p. 55.

On account of the two conservation laws mentioned above, *each term of this sum must vanish.* From this it follows that we can write

$$1+T_l=e^{2i\delta_l}, \tag{20}$$

where δ_l is real.[5] If we put Eqs. (20) and (16) into Eq. (12), we get, asymptotically,[6]

$$\psi(\mathbf{r})\sim e^{i\mathbf{p}\cdot\mathbf{r}}+f(\theta)e^{ipr}/r, \tag{21}$$

where

$$f(\theta)=(2ip)^{-1}\sum(2l+1)(e^{2i\delta_l}-1)P_l(\cos\theta). \tag{22}$$

The physical significance of δ_l becomes clear if we substitute Eq. (20) into Eq. (19) and express the result trigonometrically:

$$\psi(\mathbf{r})=\sum(2l+1)\psi_l(r)P_l(\cos\theta), \tag{23}$$

where

$$\psi_l(r)\sim e^{i(\delta_l+\frac{1}{2}\pi)}\sin(pr-\tfrac{1}{2}l\pi+\delta_l)/(pr). \tag{24}$$

The signs have been chosen so that if in Eq. (20) T_l vanishes, then δ_l vanishes, and if all δ_l are zero, the expression (19) represents an undisturbed plane wave. We see from Eq. (24) that δ_l has here its familiar interpretation as the phase shift which the interaction produces in the lth partial wave.[7]

These results have been obtained from a single iteration of Eq. (10). As we have mentioned before, the first Born approximation consists in substituting into Eq. (10) only that part of ψ which represents the incident plane wave. This approximation may be made in Eq. (17), and results in the omission of the second term of the integrand. With Eqs. (14) and (20), we can now obtain an approximate expression for the phase

$$e^{2i\delta_l}\sim 1-\pi i(2m/\hbar^2)\int_0^\infty V(r)[J_{l+\frac{1}{2}}(pr)]^2 r\,dr. \tag{25}$$

When the phases are small, this gives the useful approximation[8]

$$\delta_l\sim -\tfrac{1}{2}\pi(2m/\hbar^2)\int_0^\infty V(r)[J_{l+\frac{1}{2}}(pr)]^2 r\,dr, \tag{26}$$

and one can verify easily by substituting Eq. (25) into Eq. (19) and carrying out the sum with the aid of the relation

$$\sum(2l+1)[J_{l+\frac{1}{2}}(pr)]^2P_l(\cos\theta) = (2p/\pi)\sin(|\mathbf{p}-\mathbf{p}'|r)/(|\mathbf{p}-\mathbf{p}'|r) \tag{27}$$

that the result is exactly that which is obtained on performing the angular integrations in Eq. (11).[9]

SCATTERING AS AN INITIAL-VALUE PROBLEM

The solutions of the scattering problem which we have obtained in the previous paragraphs have all been formulated as solutions of the *boundary-value* problem of Eq. (1): What is the solution of Eq. (1) which at large distances from the scattering center reduces to a plane wave plus a radially scattered wave? But there is another formulation which is far more *anschaulich*, and which has attracted attention in recent years. This is to regard Eq. (1) as defining an *initial-value* problem: If at an initial time, t_0, I am given a function $\psi_i(\mathbf{r}, t_0)$, what will be the value at a later time t of that solution of Eq. (1) which satisfies the initial condition $\psi(\mathbf{r}, t_0)=\psi_i(\mathbf{r}, t_0)$? This is easy to solve if the term $V(\mathbf{r})$ is lacking in Eq. (1); that is, if ψ is the function which we have been denoting by ψ_0. But suppose we try to solve Eq. (3) by iteration, obtaining a series

$$\psi(\mathbf{r}, t)=\sum_{n=0}^{\infty}\int G_n(\mathbf{r}, \mathbf{r}'; t, t')\psi_0(\mathbf{r}', t')d\mathbf{r}'dt' \tag{28}$$

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[5] This fact is a special case of a very general theorem. The factor $1+T_l$ in Eq. (19) may be regarded as a special case of a transformation function which gives the amplitudes of the outgoing waves in a scattering problem in terms of those of the incoming waves. This is the so-called S-matrix, exhibited here in a representation in which its rows and columns are labeled by values of l, and in which, because there are no transitions in angular momentum, it is diagonal. The general theorem (see W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, New York, 1946), p. 46) states that the S-matrix is always unitary. Equation (20) results from the fact that if a unitary matrix is diagonal, all its elements are of modulus unity. In most textbook discussions of scattering, δ_l is assumed to be real without proof.

[6] See reference 3, p. 24.

[7] It is quite usual to describe low energy scattering phenomena in terms of the parameter $p\cot\delta_l$. (J. M. Blatt and J. D. Jackson, Phys. Rev. 76, 18 (1949).) The identity

$$\frac{1}{p}(e^{2i\delta_l}-1)=\frac{2i}{p\cot\delta_l-ip}$$

shows that this parameter completely describes scattering as long as δ_l is small enough so that the second term in the denominator is negligible compared with the first.

[8] See reference 3, p. 28.

[9] See reference 3, p. 90.

with

$$G_0(\mathbf{r}, \mathbf{r}'; t, t') = \delta(\mathbf{r}-\mathbf{r}')\delta(t-t'),$$

$$G_n(\mathbf{r}, \mathbf{r}'; t, t') = \int G(\mathbf{r}-\mathbf{r}'', t-t'')\, V(\mathbf{r}'', t'') \times G_{n-1}(\mathbf{r}'', \mathbf{r}'; t'', t')\, d\mathbf{r}''dt'' \quad (n>0). \quad (29)$$

We see that the right side involves only the free-particle wave function ψ_0, and we are now in a position to write down an expression which solves the initial-value problem. Let

$$F(\mathbf{r}, t) = i\hbar \int G(\mathbf{r}-\mathbf{r}', t-t')\psi_i(\mathbf{r}', t')\, d\mathbf{r}' \quad (t'<t), \quad (30)$$

where G is defined by Eq. (5). If $t'<t$, it is clear from Eq. (2) that $F(\mathbf{r}, t)$ is a solution of the wave equation for ψ_0.[10] Moreover, if we let t' approach t, we see from Eq. (5) that

$$\lim_{t-t'\to+0} G(\mathbf{r}-\mathbf{r}', t-t') = -(i/\hbar)\delta(\mathbf{r}-\mathbf{r}'), \quad (31)$$

so that in Eq. (30),

$$\lim_{t-t'\to+0} F(\mathbf{r}, t) = \psi_i(\mathbf{r}, t').$$

Thus Eq. (30) is seen to be the solution of the initial-value problem for ψ_0. If now we suppose ψ_i to be given in terms of $\mathbf{r}_0$ at a time t_0, Eqs. (28) and (30) give us a formula for ψ at all *later* times:

$$\psi(\mathbf{r}, t) = i\hbar \sum_{n=0}^{\infty} \int G_n(\mathbf{r}, \mathbf{r}'; t, t')\, d\mathbf{r}'dt' \times \int G(\mathbf{r}'-\mathbf{r}_0, t'-t_0)\psi_i(\mathbf{r}_0, t_0)\, d\mathbf{r}_0$$

$$= i\hbar \sum_{n=0}^{\infty} \int R_n(\mathbf{r}, \mathbf{r}_0; t, t_0)\psi_i(\mathbf{r}_0, t_0)\, d\mathbf{r}_0$$

$$= i\hbar \int R(\mathbf{r}, \mathbf{r}_0; t, t_0)\psi_i(\mathbf{r}_0, t_0)\, d\mathbf{r}_0, \quad (32)$$

where

$$R_0(\mathbf{r}, \mathbf{r}_0; t, t_0) = G(\mathbf{r}-\mathbf{r}_0, t-t_0),$$

$$R_n(\mathbf{r}, \mathbf{r}_0; t, t_0) = \int R_0(\mathbf{r}, \mathbf{r}'; t, t')\, V(\mathbf{r}', t') \times R_{n-1}(\mathbf{r}', \mathbf{r}_0; t', t_0)\, d\mathbf{r}'dt' \quad (n>0) \quad (33)$$

as long as the series (32) converges.[11]

[10] See the Appendix.

[11] The notations G and R refer to the fact that functions of this type are traditionally named after Green and Riemann. The convergence of the Born approximation has been investigated in detail by R. Jost and A. Pais, Phys. Rev. 82, 840 (1951). The series (33) has been derived by Feynman, Phys. Rev. 76, 749 (1949).

If the perturbing potential $V(\mathbf{r}, t)$ in Eq. (33) is the sum of several terms, the higher-order R_n become rather complicated. This is the situation which arises in the theory of fields, and to deal with it a technique has been developed in which the various integrals which result may be represented by a graphical scheme, from which the required expressions can be written down at once without the necessity of elaborate algebraic expansions.

Let us consider first the case in which V consists of only a single term. It is clear from Eq. (33) that $R_n(\mathbf{r}, t) = 0 (t<0)$, so that in the integral there, the range of t' is only from t_0 to t. We can visualize this situation if we imagine that the perturbation V acts to distort the wave function at some time during the interval between t_0 and t. When Eq. (33) is iterated n times, we see that the time between t_0 and t is chopped up into n ordered intervals, in each of which V can be considered as acting once. The nth term of the Born expansion (32) may be described as the contribution to ψ which is the result of n consecutive scatterings by V.

If now the particle is being acted upon by several perturbing potentials V_i, the totality of terms in Eq. (33) may be graphed as the totality of ways in which the particle may pass through the system and be scattered successively by the same or different V_i. These graphs are known as Feynman diagrams, and they do much to mitigate the enormous combinatorial complexities of higher order calculations in the theory of fields.

AN EXAMPLE—THE FIRST BORN APPROXIMATION

We have given two solutions of the scattering problem, but the physical models to which they correspond are quite different. As an illustration of their use, we shall give two derivations of the first Born approximation to the scattering of a particle by a spherically symmetrical potential $V(r)$. Equations (11) and (21) give for the amplitude of the scattered wave

$$f(\theta) = -(1/4\pi)(2m/\hbar^2)\int e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}} V(r)\, d\mathbf{r}$$

$$= -2\pi^2(2m/\hbar^2)\, V(\mathbf{p}'-\mathbf{p}),$$

where the last factor is the Fourier transform of $V(r)$:

$$V(\mathbf{p}) = (2\pi)^{-3} \int V(r) e^{-i\mathbf{p}\cdot\mathbf{r}} d\mathbf{r}. \quad (34)$$

If $V(r)$ is spherically symmetrical, then $V(\mathbf{p}-\mathbf{p}')$ will be so as well. It will be real and will depend only on $|\mathbf{p}-\mathbf{p}'|$. If the scattering is elastic, $p = p'$, and we have simply

$$|\mathbf{p}-\mathbf{p}'| = 2p \sin\tfrac{1}{2}\theta,$$

where θ is the angle between the directions of $\mathbf{p}$ and $\mathbf{p}'$.

The cross section $\sigma(\theta)d\Omega$ is defined as the number of particles per second scattered through an angle θ into the element of solid angle $d\Omega$ if the incident beam brings to the scatterer one particle per unit area per unit time. For elastic scattering, this turns out[12] to be just $|f(\theta)|^2 d\Omega$:

$$\sigma(\theta)d\Omega = (2\pi)^4 m^2 \hbar^{-4} [V(\mathbf{p}-\mathbf{p}')]^2 d\Omega. \quad (35)$$

Suppose now that we apply the expansion (32) to the same problem. Here, our physical picture is that of a wave-packet of some sort[13] which passes through the system and is scattered. Although in principle the formula (32) can be used to predict the behavior of an arbitrary wave-packet, the resulting formulas, as can easily be imagined from the amount of information which they must contain, are rather formidable, and it is easier to approach the problem of determining a simple scattering cross section more indirectly, by using the notion of a transition amplitude.

It follows from the general principles of quantum mechanics that if we have a system described by a wave function $\psi(\mathbf{r}, t)$ and wish to know what is the probability of finding the system in some state $\chi(\mathbf{r}, t)$, this probability is given by the square of the absolute value of the quantity $\int \chi^*(\mathbf{r}, t)\psi(\mathbf{r}, t)d\mathbf{r}$, which is called a transition amplitude. Suppose that ψ is given by Eq. (32).

$$A = i\hbar \int \chi^*(\mathbf{r}, t) R(\mathbf{r}, \mathbf{r}_0; t, t_0) \psi_i(\mathbf{r}_0, t_0) d\mathbf{r} d\mathbf{r}_0 \quad (36)$$

can now be read as the amplitude for a transition between an initial state ψ_i and a final one χ under the influence of the potential $V(\mathbf{r}, t)$ appearing in Eq. (33). The first term of Eq. (32) is of no interest, since it describes particles which do not interact at all with the potential, and gives no contribution to Eq. (36) if ψ_i and χ are orthogonal. In order to evaluate the first perturbed term, we must calculate Eq. (36) with

$$R_1(\mathbf{r}, \mathbf{r}_0; t, t_0) = \int R_0(\mathbf{r}, \mathbf{r}'; t, t') V(r') \times R_0(\mathbf{r}', \mathbf{r}_0; t', t_0) d\mathbf{r}' dt'. \quad (37)$$

This integral is easily done, for from Eq. (30) we have

$$\psi(\mathbf{r}, t) = i\hbar \int R_0(\mathbf{r}, \mathbf{r}'; t, t') \psi(\mathbf{r}', t') d\mathbf{r}', \quad (38)$$

whereas from Eq. (5) the complex conjugate of this is

$$\psi^*(\mathbf{r}, t) = i\hbar \int \psi^*(\mathbf{r}', t') R_0(\mathbf{r}', \mathbf{r}; t', t) d\mathbf{r}'. \quad (39)$$

These two formulas make the integral (36) very simple; taking R as R_1 for the first Born approximation, we have

$$A = (i\hbar)^{-1} \int \chi^*(\mathbf{r}', t') V(r') \psi_i(\mathbf{r}', t') d\mathbf{r}' dt'. \quad (40)$$

Suppose that the volume of the whole system is Q, which we take to be very large. Then, normalizing ψ_i so that one particle at a time is in Q, let ψ_i be $Q^{-\frac{1}{2}} \exp i(\mathbf{p}\cdot\mathbf{r} - \omega t)$ and similarly, let χ be $Q^{-\frac{1}{2}} \exp i(\mathbf{p}'\cdot\mathbf{r} - \omega' t)$, where $\hbar\omega$ and $\hbar\omega'$ give, of course, the total energies of particles in the two states. These are states of definite momentum, orthogonal to each other as long as $\mathbf{p}$ and $\mathbf{p}'$ are different, and Eq. (40) gives the amplitude for a transition between them. It follows from Eqs. (5) and (37) that the limits of the integral over t' are t_0 and t; this has an obvious physical meaning. When the integration is done, we have

$$A = -(2\pi)^3/(\hbar Q) V(\mathbf{p}'-\mathbf{p}) \times (e^{-i(\omega-\omega')t} - e^{-i(\omega-\omega')t_0})/(\omega-\omega'). \quad (41)$$

This is in general a fluctuating function of t; its square is the transition probability

$$P = 2(2\pi)^6/(\hbar^2 Q^2) |V(\mathbf{p}-\mathbf{p}')|^2 \times (1-\cos((\omega-\omega')(t-t_0)))/(\omega-\omega')^2, \quad (42)$$

[12] See reference 3, pp. 19 f.

[13] Such a wave-packet is restricted only by the requirement, sometimes forgotten, that it have no sharp corners, for with regard to the high frequency components of such a packet the present theory tells us nothing, since it is not relativistic.

which shows that there is in general a small probability of finding a particle whose energy and momentum are different from those of the initial state, the probability decreasing rather rapidly as $|\omega-\omega'|$, which gives the energy change, increases.

The possibility of finding particles with energies other than those allowed by the conservation of energy is due simply to indeterminacy—since the system has been in action only for a limited time, the energies in it are not definite. In fact, the width in $\omega-\omega'$ of the central maximum of the distribution (42) is just of the order of $(t-t_0)^{-1}$, as required by Heisenberg's relation. The total transition probability is obtained by integrating Eq. (42) over all possible values of the energy difference. Suppose that $\rho(\omega')$ is the density of possible final energy-states. The total probability is

$$P=2(2\pi)^6|\,V(\mathbf{p}-\mathbf{p}')\,|^2/(\hbar^2Q^2)$$
$$\times\int(1-\cos((\omega-\omega')(t-t_0)))\rho(\omega')d\omega'/(\omega-\omega')^2.$$

The evaluation of this integral is familiar,[14] and gives

$$P=(2\pi)^7(t-t_0)/h^2|\,V(\mathbf{p}-\mathbf{p}')\,|^2\rho(\omega), \quad (43)$$

where $\hbar\omega$ is now the energy approximately common to initial and final states. This probability will lead us directly to the scattering cross section as we have defined it above.

First we must evaluate $\rho(\omega)$. This is easily done if we start with $\rho(\mathbf{k})$, the density of states enumerated according to their wave numbers. It is well known[15] that this is $k^2Qdkd\Omega/(2\pi)^3$, where Q is the total volume of the system and $d\Omega$ is an element of solid angle in the directions of $\mathbf{k}$. We have

$$\rho(\mathbf{k})dk=\rho(\omega)d\omega=\rho(\omega)(d\omega/dk)dk=k^2Qdkd\Omega/(2\pi)^3,$$

so that

$$\rho(\omega)=k^2Qd\Omega/((2\pi)^3d\omega/dk).$$

Since ω is $hk^2/2m$, we have

$$\rho(\omega)=mkQd\Omega/(2\pi)^3h,$$

and Eq. (43) is

$$P=((2\pi)^4(t-t_0)mkd\Omega/\hbar^3Q)\,|\,V(\mathbf{p}-\mathbf{p}')\,|^2. \quad (44)$$

Since ψ and χ were normalized so as to represent one particle in the volume Q, there are hk/mQ particles per unit time per unit area arriving in the neighborhood of the scatterer. In Eq. (44) P is the cumulated probability that one particle has set out in the direction $\mathbf{p}'$, so that the number of particles scattered per unit time is $P/(t-t_0)$. The number of particles leaving the scatterer per second if *one* particle is arriving there per unit area per second is $(P/(t-t_0))(mQ/\hbar k)$, and this is what we have defined as the scattering cross section. It is the same as the expression (35), derived by the boundary-value method.

The reader should not be deceived by the apparent complexity of the initial-value as compared with the boundary-value calculation, for the latter is adapted essentially only to the exact sort of calculation for which we have used it here, whereas the former, which forms the basis of the Feynman-Dyson method in electrodynamics, is enormously more general, and can be used, for example, in the solution of more difficult perturbation problems in which the initial or final state or both are bound states, or are in some other way distorted from the plane and radial waves of the boundary-value problem.

As a final remark, it is illuminating to consider the question of measurements made on a quantum-mechanical system in the light of the series (32). The potential V appears here as the interaction of the system with the measuring apparatus, and the ideal measurement is one in which V is strong enough so that there is an appreciable number of interactions, but weak enough so that the third term of the series (32) is negligible. This corresponds to measurements in which at most one quantum of energy is exchanged between the measuring apparatus and the system under observation, so that the latter suffers the least possible perturbation. It is clearly possible to derive a quantitative specification of the ideal measurement, and we see also how such a measurement, or indeed any quantum-mechanical interaction, may be regarded as an instantaneous and unanalyzable process.

[14] See P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford University Press, London, 1947), third edition, Sec. 46.

[15] See F. K. Richtmeyer and E. H. Kennard, *Introduction to Modern Physics* (McGraw-Hill Book Company, Inc., New York, 1942), third edition, Sec. 87, leading to Eq. (109*a*).

APPENDIX

The close relation which exists between the boundary-value problem for a field with a source and the initial-value problem for a field without a source can clearly be seen if instead of Eq. (1) we solve a fictitious boundary-value problem in which a matter field φ has a source, but in which for simplicity there is no potential V:

$$(i\hbar\partial_t+(\hbar^2/2m)\Delta)\varphi(\mathbf{r},t)=\rho(\mathbf{r},t). \qquad \text{(I)}$$

The solution of this which is subject to the causality condition is

$$\varphi(\mathbf{r},t)=\varphi_0(\mathbf{r},t)+\int G(\mathbf{r}-\mathbf{r}',t-t')\rho(\mathbf{r}',t')d\mathbf{r}'dt',$$

where φ_0 satisfies the homogeneous equation corresponding to Eq. (I). This is to be compared[16] with the solution (30) of the initial-value problem for a sourceless field:

$$\varphi(\mathbf{r},t)=i\hbar\int G(\mathbf{r}-\mathbf{r}_0,t-t_0)\varphi_i(\mathbf{r}_0,t_0)d\mathbf{r}_0. \qquad \text{(II)}$$

This relation between two equations which are ostensibly quite distinct explains why some rycent work on the theory of fields[17] starts be constructing the solutions of the source problem for a field which in nature is without sources.

The function G is defined by Eq. (II) as well as it is by Eq. (3), and may be constructed without difficulty from Eq. (II). If we write the equation obeyed by φ in the form

$$i\hbar\partial_t\varphi(\mathbf{r},t)=(\mathbf{p}^2/2m)\varphi(\mathbf{r},t),$$

it is clear that a symbolic solution of it which solves the initial-value problem is given by

$$\varphi(\mathbf{r},t)=\exp\left[-\frac{i}{\hbar}\frac{\mathbf{p}^2}{2m}(t-t_0)\right]\varphi_i(\mathbf{r},t_0). \qquad \text{(III)}$$

We cannot at once evaluate the effect of operating on φ by an exponential function of $\mathbf{p}^2$, so setting $t-t_0$ equal to τ, we represent this function by the equivalent definite integral

$$\exp\left[-\frac{i}{\hbar}\frac{\mathbf{p}^2}{2m}\tau\right]=\left(\frac{m}{2\pi i\hbar\tau}\right)^{\frac{3}{2}}\times\int\exp\left[i\frac{m}{2\hbar\tau}\mathbf{u}^2+\frac{i}{\hbar}\mathbf{u}\cdot\mathbf{p}\right]d\mathbf{u}.$$

Operating with this expression on φ_i we note that $\mathbf{p}$ now appears in the combination

$$e^{i(\mathbf{u}\cdot\mathbf{p})/\hbar}=e^{\mathbf{u}\cdot\nabla}.$$

The right side of this is just the symbolic representation of a Taylor series (in three variables), as may be verified by expanding the exponential; in fact,

$$e^{\mathbf{u}\cdot\nabla}\varphi_i(\mathbf{r},t_0)=\varphi_i(\mathbf{r}+\mathbf{u},t_0),$$

so that Eq. (III) is

$$\varphi(\mathbf{r},t)=\left(\frac{m}{2\pi i\hbar\tau}\right)^{\frac{3}{2}}\int\exp\left[i\frac{m}{2\hbar\tau}\mathbf{u}^2\right]\varphi_i(\mathbf{r}+\mathbf{u},t_0)d\mathbf{u}.$$

Setting $\mathbf{r}+\mathbf{u}=\mathbf{r}'$, we find that the function G which we have constructed in this way is the same as that which we would get by performing the k-integral in Eq. (5).

[16] The question is discussed, but with special reference to equations of hyperbolic form, in R. Courant and D. Hilbert, *Methoden der Mathematischen Physik* (Verlag Julius Springer, Berlin, 1937), Vol. 2, pp. 156 ff.

[17] J. Schwinger, Proc. Nat. Acad. Sci. **37**, 452 (1951); R. P. Feynman, Phys. Rev. **84**, 108 (1951), Sec. 7.

There are in science immense numbers of different methods, appropriate to different classes of problems; but over and above them all, there is something not easily definable, which may be called the method of science. It was formerly customary to identify this with the inductive method, and to associate it with the name of Bacon. But the true inductive method was not discovered by Bacon, and the true method of science is something which includes deduction as much as induction, logic and mathematics as much as botany and geology.—BERTRAND RUSSELL, Mysticism and Logic (1918).

College Courses in Electron Microscopy

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The facilities and plans for instruction in electron microscopy in representative universities are briefly described. Recommendations are presented regarding principal instrumental and specimen techniques to be taught, laboratory equipment, student supplies, and useful publications. These recommendations evolved from an inquiry among those practicing electron microscopy in academic and industrial research laboratories.

THE electron microscope is an aid to vision in the study of solid structures in the range 0.001–10 microns. As such, and not necessarily as a specialized development of electron optics, electron microscopy can claim a place in the college training of students in the physical and biological sciences.

To assay the value of instruction in electron microscopy and the scope of present programs of instruction, all of recent inception, inquiries and a questionnaire were sent to some 150 members of the Electron Microscope Society of America and to academic and industrial electron microscopy laboratories here and abroad, particularly those which offer formal or informal instruction. Table I and the following comments on individual programs present a fair sampling of the present facilities for and scope of instruction. Colleges other than those listed have conducted courses or symposia in electron microscopy, but only those which provided information about their programs are included in the table.

The need for more research workers trained in electron microscopy is apparent in the number of positions offered. Also industrial laboratories frequently ask where they can have their personnel receive training in instrumental and specimen preparation techniques.

The value of electron microscopy as part of college education is less tangible. Several schools have curtailed or dropped their offerings in electron microscopy. These adjustments are sometimes dictated by the economic need to find a feasible balance between sponsored research and academic instruction. An electron microscope laboratory requires a large initial investment and involves a high cost per student. But generally university facilities for instruction in electron microscopy are being expanded, mostly for graduate students. Our own view is apparent in the fact that in liberalizing our undergraduate physics curriculum, in 1950, we dropped a course in electron optics for seniors but continued laboratory instruction in electron microscopy as one of several fields which a graduate student may elect in successive semesters under the course title Advanced Physical Measurements.

Persons planning new courses in electron microscopy may be interested in some recommendations about techniques to be taught, convenient sources of pertinent information, and necessary laboratory equipment and supplies. Tables II and III are composites of the techniques considered important by both industrial and academic electron microscope laboratories.

The books and manuals listed in Table IV should provide an entirely adequate working library in a field whose proliferation is recorded in an ever increasing number of reports of varying quality. This library will be found very helpful in setting up a syllabus suited to local facilities and student interests.

The selection of an electron microscope may be governed largely by the extent to which its cost and maintenance can be borne by research funds. In this country, an RCA model EMU-2D would probably be first choice. For instructional use only, the less expensive RCA model EMC-2D (no longer in production) is more versatile than model EMT, a permanent magnet instrument. The model EST-1 electrostatic microscope recently announced by the Farrand Optical Co. appears to be versatile and easily serviced. The relatively inexpensive electrostatic microscope expected to be made available soon by the General Electric Company may prove to be very well suited for the instructional laboratory.

TABLE I. Representative institutions which offer instruction in electron microscopy.

Institution	Information supplied by	Year course first offered	Course repetitions to date	Number of students to date: Total	Undergraduate	Graduate G, Faculty F, Technicians T	Fields of major interest	Type of electron microscope	Use of electron microscope: Research (percent)	Instruction (percent)	Electron diffraction equipment	Vacuum system for shadowing
Case Institute of Technology[a] (Department of Chemistry)	Allen S. Powell	1947	4	8	0	8	Chemistry 5, metallurgy 2, physics 1	RCA EMU-2A	95	5	part of EMU	RCA EMV-1
Cornell University (College of Engineering)	Benjamin M. Siegel	1950	5	60	10	50	Physics, biology	2 RCA type EMU-2B	90	10	part of EMU	two, homemade
Illinois Institute of Technology[b]	Paul L. Copeland	1945	4	50	50	0	Physics 38, metallurgy 7, chemistry 3, biology 2	RCA types B and EMC	95	5	part of B	one
Indiana University[c] (Chemistry Department)	Robert B. Fischer	1948	6	18 lab 93 lect 25 audit	15	78	Chemistry, zoology, bacteriology, geology, physics	RCA EMU	85	15	part of EMU	one, homemade
Institut Pasteur,[d] Paris	Pierre Lépine	1945	continuous	50	0	34 G, 16 T	Biology	CSF electrostatic, 60 kv Siemens magnetic, 85 kv RCA EMU	67	33	none	three
Institute of Technology,[e] Delft, Netherlands (Electron Microscope Department)	J. B. le Poole	1951	2	10	3	7 G	All	1 Philips 1 homemade 1 400 kv	90	10	none	two
Karolinska Institutet,[f] Stockholm (Department of Anatomy)	Fritiof Sjöstrand	1947	3	60	60	0	Biology	RCA EMU-2C	90	10	part of EMU	one
Pennsylvania State College[g] Division of Minerology	T. F. Bates	1949	3	9	0	9 G	Mineralogy, ceramics	RCA types EMU & EMT	90	10	RCA EMD	one, homemade
Department of Physics	Robert L. Weber	1945	6	33	0	33 G	Physics 20, bacteriology 6, mineralogy & ceramics 7	RCA EMC	10	90	none	one, homemade
University of California[h] (College of Engineering)	R. S. Mackay	1948	7	400	200	160 G, 40 F	All	2 RCA EMU 1 RCA B	75	25	part of EMU	several
University of Cambridge,[i] England (Board of Extra-Mural Studies)	V. E. Cosslett	1948	3	90	0	40	Biology, metallurgy	Metropolitan-Vickers RCA EMB Siemens	95	5	separate, and as part of E.M.	two, homemade
University of Illinois[j] (Chemistry Department)	Ralph A. Johnson G. L. Clark A. E. Vatter, Jr.	1946	6	100	0	100 G & F	Ceramics, agronomy, plant pathology, botany, physics	RCA types EMB, EMC, and EMT	95	5	none	one
University of Kentucky[k] (Bacteriology Department)	O. F. Edwards	1947	8	58	0	58	Bacteriology, chemistry, metallurgy	RCA EMU-2A	50	50	part of EMU	RCA EMV-1A
University of Texas[l] (Electrical Engineering Department)	Leland L. Antes	1945	5	100	60	40	Chemistry, chemical engineering, bacteriology, ceramics	RCA EMU	50	50	part of EMU	RCA EMV
University of Toronto[m] (Physics Department)	G. David Scott	1944	8	30	0	30	Physics, metallurgy	RCA EMU-2A Toronto 1944	95		part of each E.M.	homemade
University of Wisconsin[n] (College of Agriculture)	Paul Kaesberg	1951	1	15	0	15 G	Protein chemistry	RCA EMU-2A	90	10	part of EMU	one

(Footnotes to Table I are on page 303.)

A shadow casting unit is an essential part of the electron microscope laboratory. Complete commercial units are available, but it is easy to assemble the necessary components. A small bell jar, very fast diffusion and mechanical pumps, and at least three pairs of electrodes in the base plate are desirable features. It should be possible to isolate the diffusion pump by valves so that it need not be cooled and opened to air each time the bell jar is lifted. In a busy laboratory, the shadowing unit may be a bottle neck, in which case two units may be justifiable. This facilitates shutting down one unit for cleaning or for replacing contaminated oil.

A light microscope is essential to the preparation of specimens, the interpretation of micrographs, and sometimes the maintenance of the electron microscope. A binocular microscope, with magnifications about 10×, 30×, and 90×, is especially helpful to a beginner in that he can observe the effects of specimen manipulation with right being right and top being top in the magnified image. For advanced work one might require a universal light microscope which provides bright field, dark field, "Ultrapak" or vertical illumination, and phase contrast. Phase contrast provides the only, if partial, control on the introduction of artifacts in biological specimens.

A photographic darkroom with an enlarger, printer, timer, and the usual equipment for processing plates and prints is a necessary adjunct. The enlarger should have sufficiently good definition to permit useful enlarging of at least 10×.

An electron optical bench for demonstrating and testing electron lenses would be an asset in a laboratory stressing fundamental electron optics. Its use is not at all necessary for the development of a good electron microscopist.

TABLE II. Instrumental techniques (in order of importance).

1. Testing performance
   - *a* elementary checking of vacuum tube circuits and current regulators
   - *b* care of vacuum systems
   - *c* safegarding electron microscope from external influences
2. Focusing
3. Alignment
4. Determination of magnification
5. Measurement of distortion (for particle size work)
6. Photography
7. Stereoscopy
8. Determination of resolution—best considered as part of the interpretation of micrographs.

TABLE III. Specimen techniques (importance depends on application).

1. Preparation of films
2. Replica making
3. Particle dispersion and mounting
4. Shadow casting
5. Centrifugation
6. Air-spraying, for absolute particle counts
7. Thin sectioning.

Footnotes to Table I.

[a] Case Institute of Technology offers a 3-credit lecture course for undergraduates, using Wyckoff's textbook. Dr. Powell remarked that while many students are interested in learning the manipulation of the electron microscope at the level of a laboratory technician, few are interested in a fundamental study. Laboratory work is confined to graduate students' thesis problems. Research is chiefly in high polymers. The laboratory has a stereomicroscope, metallographic microscope and metallographic preparation equipment.

[b] The Illinois Institute of Technology is one of the few laboratories which has an electron optical bench for student experiments with electron lenses.

[c] Indiana University offers a lecture demonstration course and a separate laboratory course, using Fischer's textbook. Both courses stress applications rather than physical aspects of electron optics. Laboratory work is on an individual basis with a maximum of five (preferably two or three) students using the laboratory at one time.

[d] The Institut Pasteur is essentially a research institution, but it has trained all of the French biological electron microscopists and many from abroad. Advanced training is given to workers, not more than two at a time, in virus diseases, histology, and protein structures. One electron microscope is available for their exclusive use. Light and phase contrast microscopes and ultrafine sectioning equipment is available.

[e] The Institute of Technology at Delft treats all applications of electron microscopy. The laboratory has high power optical, metallurgical and preparation microscopes and an electron optical bench.

[f] The Karolinska Institutet stresses biological electron microscopy and offers instruction in binding forces, levels of organization, relations between properties and structure on a supramolecular level, specific problems regarding muscle, nerve, and connective tissue, and the techniques of freeze-drying and fixation of tissue cells.

[g] At the Pennsylvania State College, in the Division of Mineralogy, each student of electron microscopy is assigned a problem and he applies feasible methods toward its solution. In the Physics Department, a 1-credit course of 3 hr per week presents elementary electron optics (using Cosslett's textbook) and considerable practice in general methods of specimen preparation and photography. Sections are limited to four students.

[h] The University of California offers a 1-unit course comprising 3 laboratory periods and 12 lectures per semester. The chief topics are limitations on all microscopes, types of electron microscopes, magnification calibration, vacuum and photographic techniques, sectioning, replica making, shadowing, and attainment of high resolution.

[i] The University of Cambridge has twice conducted an 11-day summer school in electron microscopy, similar to their summer school in x-ray crystallography. The instructional program arranged by the Electron Microscope Group of the Institute of Physics consisted of lectures, demonstrations, and practical classes covering the fundamental theory of the electron microscope and the methods used in its principal applications. In 1950, seven lecturers and some eight specialist demonstrators presented the various aspects of electron microscopy.

[j] The University of Illinois emphasizes chemical and biological applications. Four types of course have been offered: (1) seminar for research men, without credit; (2) seminar for graduate students, 1 credit; (3) as part of a course in chemical microscopy; and (4) laboratory instruction concentrated in 3–5 days during vacation periods.

[k] The University of Kentucky offers a semester course of two 2-hr laboratory periods and one 1-hr lecture per week for graduate students, chiefly in bacteriology.

[l] The University of Texas found that several separate courses seemed necessary to meet the needs of students with different backgrounds: one for physicists and engineers, another for chemists, etc., and a course in techniques for biologists. They used a wedge tank in the study of lenses and a centrifuge for biological specimens. This program has been discontinued.

[m] The University of Toronto has offered a laboratory course in electron microscopy only once (1945). The lecture course in electron optics and electron microscopy was originally 2 hr per week, but over the past four years has been 1 hr per week.

[n] The University of Wisconsin has devoted 2 weeks of a course in the Physical Chemistry of Proteins to electron microscopy. Topics emphasized were: (1) methods for determining size, shape, molecular weight and hydration of proteins and viruses; (2) preparative techniques useful for macromolecules and fibrous proteins: staining, shadowing, fragmenting, freeze drying, sectioning, etc.; and (3) properties of microscopes: resolving power, distortion, contrast, etc. At least eight other departments have single demonstration lectures on electron microscopy in one of their classes. There is no laboratory course in electron microscopy.

Likewise the need for electron diffraction equipment is not great. Electron diffraction contributes little to electron microscopy, though evidence from electron microscopy is highly important in electron diffraction studies.

Two centrifuges are useful in specimen mounting as well as in the preparation of specimen materials. One should be of small angle, giving 2000*g*, the other a small, high-speed centrifuge giving up to 15,000*g*. For much biological work, a specimen freezing unit and an ultra-thin sectioning microtome are desirable.

For electron metallography, a mounting press and metallographic polishing wheel make it possible to study a wide variety of polished surfaces.

In the study of particles and fibers, the Robley Williams type of sprayer and the Waring Blender will be useful occasionally.

The foregoing list of major accessories may seem formidable. Fortunately any one experimental program will not require them all. The smaller accessories and supplies needed for instruction are listed in Table V. The items italicized are most essential and might be assembled in the form of kits for each student or group of students. Given this kit, a properly tuned up electron microscope, a light microscope, and a shadowing unit, student and instructor have good facilities to embark on fascinating explorations with the electron microscope.

I wish to acknowledge with gratitude the stimulating help received from several persons in addition to those named in Table I: C. J. Calbick, Joseph J. Comer, W. L. Grube, James Hillier, William R. McMillan, Stuart Mudd, D. Gordon Sharp, John D. Steely, and Norman M. Walter.

TABLE IV. A book shelf for the electron microscopy laboratory.

W. G. Berl, *Physical Methods in Chemical Analysis* (Academic Press, Inc., New York, 1950). Volume 1 contains "Electron Microscopy" by R. D. Heidenreich, pp. 535–586.

B. von Borries, *Die Übermikroskopie* (Aulendorf/Württ, Cantor, 1949). Good bibliography.

E. M. Chamot and C. W. Mason, *Chemical Microscopy* (John Wiley & Sons, Inc., New York, 1938), 2d ed., vol. 1.

V. E. Cosslett, Ed., *Bibliography of Electron Microscopy* (Edward Arnold & Company, London, England, 1950).

V. E. Cosslett, *Introduction to Electron Optics* (Oxford University Press, New York, 1950), 2d ed.

V. E. Cosslett, *Practical Electron Microscopy* (Academic Press, Inc., New York, 1951).

D. G. Drummond, Ed., "The practice of electron microscopy," J. Roy. Microscop. Soc. **70**, 1–158 (1950).

Eastman Kodak Co., *Kodak Reference Handbook* (Eastman Kodak Co., Rochester).

R. B. Fischer, *Applied Electron Microscopy* (Indiana University Duplicating Dept., Bloomington, Ind., 1950), 144 pp, mimeographed.

C. Grégoire, *Microscope électronique et Recherche biologique* (Préf. d'Albert Claude. Liège, Desoer, 1950). Médecine et Biologie no. 9.

E. Leitz, Inc., *The Microscope and Its Application* (E. Leitz, Inc., 1938), or a similar handbook for the light microscope available.

A. A. Rusterholz, *Elektronenoptik* (Birkhäuser, Basel, 1950); Lehr- und Handbücher der Ingenieurwissenschaften, 15.

R. W. G. Wyckoff, *Electron Microscopy, Technique and Applications* (Interscience Publishers, Inc., New York, 1949).

V. K. Zworykin, *et al.*, *Electron Optics and the Electron Microscope* (John Wiley & Sons, New York, 1945).

NOTE: Almost every laboratory has a set of the bibliography cards sponsored by the Electron Microscope Society of America. But when queried about the extent to which they used the cards, informants replied "little" or "none."

TABLE V. Kit of instruments and supplies for the preparation of specimens in electron microscopy.

2 *Bottles*, dropping, 60 ml capacity
Box for microscope slides, 25 capacity
6 Bulbs, rubber, 1 ml capacity
Capsules, gelatine, size 00
Collodion solution (Parlodion) 2 percent and 0.2 percent in amyl acetate, freshly made. Collodion makes weak films after about six weeks.
Crucible tongs (Aloe Co. no. 80010)
Dish, petri
Forceps, blunt tipped (Aloe 37552)
Forceps, fine tipped (Aloe 37442), or jewelers forceps carefully sharpened
Forceps, spade tipped, to handle cover glass
Formvar (Polyvinyl formal) 2 percent and 0.2 percent in ethylene dichloride, freshly made. Not as versatile as Collodion. Since the two are soluble in different liquids they may be used together and one removed at will.
Glass tubing, 5-mm inside diameter
Micropipettes, homemade, drawn from 5-mm glass tubing, fitted with small rubber bulbs
Pencil, glass marking
Plates, photographic, Eastman Medium Lantern Slide
Scalpel, with removable blades (Aloe 37780 & 37760)
Scotch tape
Paper, bibulous
Pin-vise and sharp dissecting *needle*, spear tipped or pointed
Polyvinyl alcohol, 10 percent in water
Paper, lens tissue
Paper, filter
Punch, for cutting ⅛-in. diameter specimen screens
Scissors, small surgical, or B & S 140 mm (Aloe 38480)
Slide holder (P. Kaesberg recommends a glass rod bent at right angles and fitted with a small rubber suction cup for handling microscope slides when picking up films cast on water.)
Slides, microscope, 1×3 in.
Spatula, stainless steel
Specimen screens, 200 mesh copper Electromesh have better mechanical and thermal stability than do woven screens. Punched ⅛-in. disks are used for specimen carriers. Larger pieces are used to handle replicas, sections, and particles dispersed in film. Screens are cleaned just before using by a light etch in nitric acid, followed by careful washing. (In ordering screens, specify 25% open area.)
Watch glasses, 1½-in. diameter.

NOTES AND DISCUSSION

A Wave-Motion Slide Rule

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PERIODIC motion is a common element in a large percentage of the physical phenomena observable in nature. In many cases this motion gives rise to a transfer of energy by wave motion in the medium surrounding the vibrating system. It is this very transfer of energy, as in the case of sound, that makes our experimental observation possible. It is therefore important that the elementary physics student understand, at least in a simple geometrical form, the nature of wave motion resulting from the related periodic motions of many particles in a medium. The motion of a single particle, such as the simple harmonic motion executed by a mass on the end of a spring, may be represented in two dimensions by plotting the displacement of the mass *versus* the time. However, in sine-wave motion in a medium produced by a simple harmonic vibrator source, the student is interested in the simultaneous displacements of many particles for which the displacement y is a function of both the position x of the particle relative to the origin and the elapsed time t since the start of the disturbance.

The equation of such a progressive wave is given by

$$y = A \sin 2\pi[(t/T) - (x/\lambda)],$$

where A is the amplitude, T the period of the vibration, and λ the wavelength. This equation is often represented in two dimensions either by fixing t and examining the displacement of all the particles in the medium at that instant or by fixing x and plotting the simple harmonic displacement of that single particle as a function of time. Both of these relationships are sinusoidal, and because of their similarity in appearance the unwary student often fails to see clearly the difference between them. A common error resulting from this confusion is that the student is apt to plot both time and distance on the abscissa. Students making this error are likely to use wavelength and period synonymously.

To aid in visualizing the meaning of this equation the author has devised a wave-motion slide rule which allows the student to examine the displacement of all the particles in the medium at a given instant or to examine the displacement of a single particle a fixed distance from the origin as a function of time. The slide rule (Figs. 1 and 2) consists of a sinusoidal wave form which is free to slide horizontally in front of a grid. A mask, in which there is a large rectangular opening, is fastened in front of the grid providing a space between the mask and the grid through which the wave form may slide. The large rectangular hole in the mask exactly frames the grid, such that the lower and left-hand edges of the opening form the abscissa and ordinate, respectively, for a graph. The displacement y is plotted vertically in units of $\frac{1}{10}$ of the amplitude, ranging in values from $-A$, corresponding to a maximum negative displacement at the bottom, to $+A$ or maximum positive displacement at the top. The distance scale x is plotted horizontally starting with zero at the lower left and increasing to the right in units of $\lambda/10$. A smaller opening in the mask at the lower right is provided as an indicator window for a time scale which is printed on the wave slide. This scale starts with $t=0$ at the extreme right end of the slide and increases to the left in units of $T/10$. The position of the front of the sinusoidal wave pattern, indicated by the position of the white arrow in Figs. 1 and 2, as well as the position of the time scale, has been adjusted so that both the vertical displacement of the particle at the origin and the distance traveled by the wave disturbance are zero arbitrarily when $t=0$. The student will observe that the front of the wave disturbance travels a distance measured in wavelengths which is numerically equal to the elapsed time measured in periods.

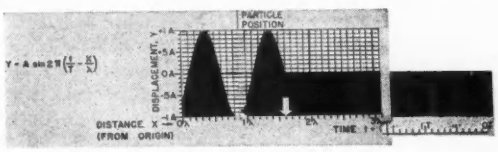

FIG. 1. Photograph of the wave-motion slide rule showing the appearance of a wave disturbance produced by a simple harmonic vibrator 1.6 periods after the start of the disturbance.

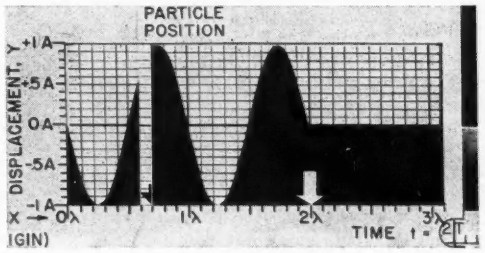

FIG. 2. An enlarged view of the slide rule showing the same disturbance $0.4T$ later than in Fig. 1. The displacement of a particle at $x=0.7\lambda$ at that instant is $0.95A$.

As the wave slide is slowly pulled out to the right, the observer is given an impression of a simple harmonic wave motion disturbance created by a particle at $x=0$ and traveling to the right toward increasingly larger values of x. If the slide is then stopped, the impression is similar to a photograph of a wave in which the shutter was snapped at the instant indicated on the time scale.

To help the student focus his attention more closely on the motion of a single particle, a "particle-position" indicator has been provided which may be placed at any arbitrary value of x. Here the student may imagine the particle to be located at the intersection of the top edge of the wave slide and the right-hand edge of the indicator blade.

In Fig. 1 the wave disturbance has traveled 1.6 wavelengths in 1.6 periods. The student can easily see that the

particle at a distance λ from the origin is in phase with a particle at the origin as indicated. Both particles are moving downward and at this instant are displaced $-0.59A$. Figure 2 shows what the wave looks like at $t=2T$, which is 0.4 of a period later than in Fig. 1. Again the vertical displacement of the particle at a distance λ from the origin is the same as the displacement of the particle at the origin. This would indicate to the student that phase differences between particles vibrating with a constant frequency in a homogeneous isotropic medium depend only upon the difference in the distance from the origin of the particles and not at all upon the elapsed time. At this instant of time the vertical displacement of both particles is zero, and both particles are moving in an upward direction, starting a new cycle. At the same instant the displacement of another particle at a distance 0.7λ from the origin, for example, is $0.95A$ as shown by the particle-position indicator. The student may readily verify these results by substituting the appropriate numerical values of x and t into the formula.

Some aspects of the Doppler effect may be demonstrated on the slide rule by imagining an observer to be riding on the particle-position indicator which is then made to move either toward or away from the origin while the wave slide is pulled out at a constant velocity. The apparent difference in frequency from that of the source is graphically demonstrated.

TABLE II. A list of the starred physicists who had published 51 or more papers up to 1943, arranged in order of total papers published.

Starred physicist	Papers published up to 1943	Total working years up to 1943	Papers per year
R. W. Wood	182	45	4.0
W. W. Coblentz	139	40	3.5
H. E. Ives	123	35	3.5
O. W. Richardson	121	39	3.4
P. W. Bridgman	117	35	3.3
W. F. G. Swann	108	33	3.3
R. A. Millikan	106	45	2.4
A. Einstein	96	45	2.1
R. C. Tolman	84	35	2.4
W. F. Meggers	83	26	3.2
E. O. Hulburt	81	28	3.0
G. Breit	81	22	3.7
E. Fermi	74	21	3.5
A. H. Compton	73	27	2.7
K. T. Compton	73	31	2.4
A. S. King	67	40	1.7
R. Ladenburg	66	37	1.8
R. S. Mulliken	66	22	3.0
H. A. Wilson	65	43	1.5
K. F. Herzfeld	64	29	2.2
A. Landé	63	28	2.3
L. B. Loeb	62	27	2.3
F. L. Mohler	61	26	2.4
P. D. Foote	59	27	2.2
J. Franck	58	36	1.6
J. H. Van Vleck	58	21	2.8
H. A. Bethe	56	14	4.0
W. E. Forsythe	56	31	1.8
I. S. Bowen	54	18	3.0
W. J. Humphreys	54	45	1.2
W. Pauli	54	22	2.5
C. G. Abbott	53	44	1.2
M. Luckiesh	53	34	1.6
L. W. McKeehan	53	33	1.6
L. N. Brillouin	52	22	2.4
G. E. M. Jauncey	52	33	1.6
F. Zwicky	52	21	2.5
A. L. Hughes	51	36	1.4

A Comparison of the Publication Records of Starred Physicists and Chemists

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THERE exists a numerical compilation[1] of the scholarly publications of the 250 physicists starred in the 1944 edition of *American Men of Science*. It has now been possible to assemble similar data on the 263 chemists starred in the same edition.[2] In this study of the literary production of chemists a procedure exactly similar to that for the physicists was employed. As before, the names of the starred chemists were found through a page-by-page search, since no listing is available. For each man the annual volumes of *Chemical Abstracts* were then consulted in order to compile a listing of the number of papers, including notes and letters to the editor, abstracted for each man for each year from the time that man received his Ph.D. degree up to and through 1942. Further details of the procedure are given in the first reference.

TABLE I. Papers published per starred Ph.D. graduate per year, grouped by institutions.

Institution	Number of Ph.D. graduates starred as physicists	Papers per man-year by starred physicists	Number of Ph.D. graduates starred as chemists	Papers per man-year by starred chemists
Munich	5	1.9	2	—
California	6	1.7	22	2.9
California Institute	12	1.6	7	3.7
Wisconsin	6	1.2	7	2.7
Harvard	15	1.0	29	3.5
Yale	9	1.0	13	2.4
Princeton	20	0.96	8	3.2
Minnesota	10	0.92	2	—
No doctorate	19	0.92	27	2.1
Chicago	18	0.88	16	3.3
Columbia	12	0.87	9	5.5
Michigan	7	0.84	5	3.9
Johns Hopkins	32	0.81	15	1.5
Göttingen	6	0.80	8	2.6
Cornell	16	0.75	6	2.8
Clark	6	0.21	0	—
Illinois	1	—	11	3.0
Massachusetts Institute	3	—	8	2.9
Brown	0	—	7	2.7
Leipzig	4	—	13	2.5
Pennsylvania	2	—	8	2.1

Clearly, such a compilation is not a generally acceptable measure of the contribution a scientist makes to his chosen field, and we do not propose to suggest that the listings to follow are intended to show the relative worth of the work of the men listed.

In Table I the starred physicists and chemists have been grouped under the names of the institutions where they received their Ph.D. degrees. Only those institutions granting the degree to five or more physicists or chemists are included. Without making a statistical study it can be readily seen that there is no highly significant correlation between the number of starred physicists and the corresponding number of starred chemists graduated by the individual schools. That is to say, an institution may have graduated a relatively large number of physicists who became starred, but not necessarily have graduated a large number of chemists. Thus Johns Hopkins has graduated the most starred physicists while Harvard has graduated the most starred chemists. Similarly, there does not appear

TABLE III. A list of the starred chemists who had published 140 or more papers up to 1943, arranged in order of total papers published.

Starred chemist	Papers published up to 1943	Total working years up to 1943	Papers per year
Gustav Egloff	552	27	20.4
Isaac M. Kolthoff	512	25	20.5
Henry Gilman	354	25	14.2
Roger Adams	338	31	10.9
V. N. Ipatieff	332	36	9.2
Treat B. Johnson	322	36	8.9
Max Bergmann	309	32	9.7
Samuel E. Shepard	294	36	8.2
Edwin B. Hart	263	36	7.3
Wilder D. Bancroft	260	36	7.2
Leonor Michaelis	255	36	7.1
Harold Hibbert	250	36	6.9
Herman F. Mark	242	22	11.0
Hugh S. Taylor	231	29	8.0
William D. Harkins	229	35	6.5
James W. McBain	219	35	6.3
Donald D. Van Slyke	214	36	5.9
Louis F. Fieser	213	19	11.2
Marston T. Bogert	211	34	6.2
Carl S. Marvel	209	23	9.1
Colin G. Fink	207	36	5.8
Walter A. Jacobs	204	36	5.7
Arnold C. Fieldner	199	36	5.5
C. S. Hudson	192	36	5.3
H. C. Sherman	188	36	5.2
E. Emmett Reid	183	36	5.1
Ralph H. McKee	182	36	5.1
Raymond F. Bacon	179	36	5.0
Gregory B. Baxter	171	36	4.8
Irving Langmuir	171	36	4.8
Frank C. Whitmore	171	29	5.9
George L. Clark	166	25	6.6
Morris S. Kharasch	159	24	6.6
Homer B. Adkins	156	27	5.8
Kenneth C. Hickman	149	18	8.3
Charles A. Browne	147	36	4.1
E. G. Benedict	145	36	4.0
Arthur M. Buswell	143	26	5.5
Ernest A. Hauser	142	21	6.8
Ralph G. Wyckoff	140	24	5.8

to be a high correlation between the relative average productivity in number of papers per year of the starred Ph.D.'s in physics from an institution and the corresponding number for starred Ph.D. graduates in chemistry. Thus California and the California Institute of Technology physics graduates published at a greater annual rate than did the physicists graduated by other American institutions, while the graduates of Columbia excelled among the chemists.

It is easily noted, however, that as a group the chemistry graduates publish more papers each year on the average than do the corresponding physics graduates from each institution, without exception. Indeed, compared as a group, including starred men whose *alma maters* are not listed in Table I, the chemists have been publishing almost exactly three times as many papers per working year as have the physicists, man for man. The 250 starred physicists published 0.93 papers on the average per man per year; the 263 starred chemists 2.87.

The names of the physicists who have published extensively over a number of years are given in Table II and a similar list of chemists may be found in Table III. To be seen again is the markedly higher literary productivity (by counting of papers) of the chemists. Whereas the leaders among the chemists, Gustav Egloff and Isaac M. Kolthoff had, in 1943, published over 500 papers each, R. W. Wood had published 182 articles to lead the starred physicists. In terms of time-rate of publication, the top chemists published on the average 20 papers per year; the top physicists 4 per year. In one year (1940) Egloff published 53 papers and patents, some with co-authors. In contrast, the leaders in this respect among the physicists were R. W. Wood and H. E. Ives with 13 each in particular years.

It is of interest to mention the record of Lord Kelvin in the way of number of publications. During his lifetime he published, according to one biographer,[3] 661 papers, some of which appeared with little or no change in as many as four different publications, some 70 patents, and 25 books and pamphlets. Living to be 80 years of age, he published his first paper in 1841 and from then to 1907, the year of his death, there was no year without the appearance of a paper. In 1875 he had 24 articles and abstracts to his credit. So far as I can determine, this publication record is unmatched by any other physicist before or since. The only comparable record[4] seems to be that of Sir Oliver Lodge, who published 1156 works from 1875 to 1935, but these included material on psychical research, philosophy, and religion. His papers and books on electricity, radio, and the ether numbered about 600.

[1] R. T. Lagemann and B. E. K. Alter, Jr., Am. J. Phys. **16**, 96 (1948).
[2] The 1944 edition of *American Men of Science* is the latest one available in which a star or asterisk is used to designate special eminence. The practice is discontinued in the 1949 edition.
[3] Sylvanus P. Thompson, *Life of Lord Kelvin*, Vol. II, London, 1910.
[4] Theodore Besterman, *A Bibliography of Sir Oliver Lodge* (Oxford University Press, London, 1935).

Microwave Demonstrations*

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(Received January 14, 1952)

THE importance of microwaves in the teaching of physical optics has been pointed out by Andrews[1] using 2500-megacycle (12-cm) commercial equipment.

Persons interested in constructing rather inexpensive microwave equipment (420 megacycles) that may be used for this purpose may find the circuits and description of this type equipment which appeared in a recent issue of *Ham News*[2] of interest.

Two 420-megacycle oscillators, together with antenna, intensity meter, and Lecher-wires are shown in Fig. 1. This apparatus may be used to demonstrate several properties of electromagnetic waves, such as interference, reflection, polarization, standing waves, etc. It may also be used in the teaching of microwave transmission as applied to radio and television.

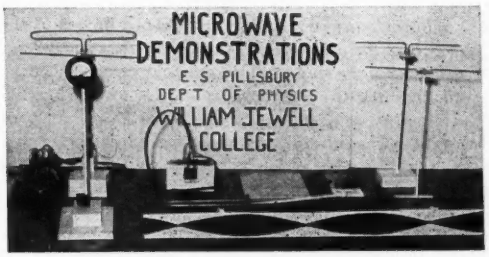

FIG. 1. Microwave demonstration apparatus.

Experiments that may be performed include the following:

1. Reflection may be studied using a metal reflector or a half-wave wire reflector.
2. Radio fading may be shown using a reflector to represent the ionosphere.
3. Standing waves may be shown by moving the intensity meter between the antenna and a reflector.
4. Polarization may be shown by rotating the intensity meter, or by the insertion of a parallel wire screen between the antenna and intensity meter.
5. Field pattern of an antenna may be observed by reading the intensity meter at various locations.
6. Focusing of electromagnetic waves may be demonstrated with a half-wave reflector.
7. The effect of a reflector on a television antenna may be shown by using a half-wave reflector with the intensity meter.
8. Using a Lecher-wire system, standing waves may be demonstrated and the frequency of the oscillator measured.

* Exhibited at the 12th Annual Colloquium of College Physicists State University of Iowa, June 13–16, 1951.
† Now in United States Navy.
[1] C. L. Andrews, Am. J. Phys. **14**, 379–82 (1946) and **17**, 462 (1949).
[2] Ham News, "Table Top Antennas," **4**, No. 1, 1–5, Jan.–Feb. (1949). General Electric Company, Schenectady, New York.

Inversion of the Retinal Image

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THE inversion of the retinal image can be shown to a class by the well-known method of having them look at an extended source or open window through a pinhole held about 2 or 3 inches in front of one eye. A pencil is held up against the observer's spectacle lens, or roughly that much in front of his pupil, and moved back and forth across the field of view. An inverted shadow of the tip is seen moving in the opposite direction to the motion of the pencil. A ray diagram can be used to clarify what happens. The explanation goes thus: The pinhole is too close to the eye for accommodation and acts only as a stop; it permits essentially only one ray from each point of the window or source to form the retinal image but, of course, does not affect the inverted character of the latter. The pencil point intercepts rays going to the bottom of the retina, casting an inverted V-shaped shadow. The brain has learned by past years of conditioning to associate such a retinal stimulus with the existence of an upright V-shaped object in the upper part of the field of view, and this is what one "sees."

Incidentally, the student will agree that it is really not so remarkable that we see upright objects from inverted retinal images if he thinks of the retina as an array of photocells, each with its own connection (optic nerve) to a remote sensitive relay or detector (brain cell). The essential point is that the pattern of thrown relays has a 1–1 correspondence to the pattern of illuminated photocells, and, in turn, to the structure of the external physical object doing the illuminating; this correspondence is established by each of us at an early age. However, the specific geometrical correspondence between the object and the photocells is irrelevant; we would probably have learned to see just as well if our eyes formed horizontal images of vertical objects, or if the images formed were grossly distorted, so long as the 1–1 correspondence were not disturbed. Similarly, if by some hypothetical accident at birth, an infant's eyeballs were turned through 180° so that nasal and temporal sides were interchanged but the eye and optic nerves were not damaged, he would grow up with normal vision, but, after he had learned to see this way, an operation that caused the eyeballs to revert to normal position would make everything appear upside down.

Some time ago, I noticed that slight pressure with the tip of a finger at the edge of an eyeball (with the eyes closed) caused a luminous sensation to appear at a point diametrically across from the finger, and, as the finger position changed, the glow moved accordingly. One can oscillate his finger tip slowly and notice the out-of-phase oscillation of the image. Sometimes, one eye seems more responsive than the other. The effect is observable either in light or darkness. Apparently, pressure and other nonradiant stimuli can cause retinal response ("seeing stars"); here pressure on the upper right part of the retina causes a sensation but it appears in the lower left part of the field of view as the brain interprets it. This explanation is put forth tentatively. It is not clear why the applied pressure does not affect the whole retina equally; perhaps it is more a matter of distortion than of pressure. A detailed explanation for the retina's response to pressure would be desirable.

A Conservation Theorem in Classical Mechanics*

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IN trying to teach students some of the limitations implicit in the usual formulations of the energy-integral and Lagrange-equation methods of solving mechanics problems, it may be of value to start from a conservation theorem which is not usually presented in the textbooks on such subjects. The purpose of this note is to present this theorem.

Newton's second law is written in the form

$$\frac{d}{dt}(m\mathbf{v})=\mathbf{F}.$$

Take the scalar product of each side with the quantity $m d\mathbf{r}$, giving

$$m d\mathbf{r}\cdot\frac{d}{dt}(m\mathbf{v})=\mathbf{F}\cdot m d\mathbf{r}.$$

The usual rearrangement of differentials results in

$$m(d\mathbf{r}/dt)\cdot d(m\mathbf{v})=m\mathbf{v}\cdot d(m\mathbf{v})=\mathbf{F}\cdot m d\mathbf{r},$$

which, integrated, gives the desired conservation theorem

$$\tfrac{1}{2}(m\mathbf{v})^2-\int\mathbf{F}\cdot m d\mathbf{r}=\text{constant}.$$

In this formulation it has not been assumed that the mass is constant so that this theorem is more general than the usually found theorem derived in this manner. From this

theorem the usual conservation of energy law is obtained by assuming the mass is constant.

This formulation allows easy attacks on the standard varying-mass problems, e.g., calculation of the maximum height attained by an upward-projected ball to which is attached a long rope of prescribed mass per unit length. Note that it would still be easy to handle the case where the mass per unit length varies with the length.

By approaching conservation theorems from this point of view, the student may become more cognizant of the assumptions underlying the more specialized laws of conservation of momentum and energy. Further, since the momentum appears in this formulation, it receives an emphasis probably deservedly greater than that accorded the kinetic energy. Also, some of the formulations in relativistic mechanics may seem less strange.

Finally, a rearrangement under the integral sign leads to another form of the theorem

$$\frac{(m\mathbf{v})^2}{2}-\int \mathbf{F}\cdot m\mathbf{v}\frac{d\mathbf{v}}{dt}dt=\frac{(m\mathbf{v})^2}{2}-\int \mathbf{F}\cdot m\mathbf{v}dt=\text{constant}.$$

In this form, for example, it is easily seen that if the applied force is always perpendicular to the momentum, the magnitude, though not the direction, of the momentum remains constant.

* Publication number 6 of the Department of Biophysics, University of Pittsburgh.

A Flame Source for Spectroscopy

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THE production of simple line spectra for use in an elementary laboratory is a rather difficult problem. Arc sources and discharge tubes are generally unsatisfactory for this type of work. The arc spectra are so complex that the student of elementary physics is usually quite bewildered by the multiplicity of observed lines. Discharge tubes are limited to gases only; they are expensive, short-lived, and in some cases dangerous to operate because of the high voltage involved. Flame spectra are usually simple enough that the student is able to measure wavelengths of the few bright lines with sufficient accuracy to identify "unknown" metals. Various devices are commercially available which consist either of metallic-salt beads or of asbestos wicking moistened with salt solutions by capillary action. The use of beads for elementary laboratory is unsatisfactory because of the low intensity and the relatively short life of the beads. The standard wick devices do not in general produce spectra which are sufficiently brilliant for usual observation especially with grating instruments. The inexpensive device shown in Fig. 1 has been used at Lafayette College and was found to be extremely satisfactory. It consists of a twelve-inch piece of one-half inch copper tubing bent in a smooth right angle and supported in a vertical plane. The horizontal portion of the tube is packed with asbestos wicking which protrudes about one inch; the vertical portion is filled with a strong solution of the salt whose spectrum is to be investigated. Under the influence of its own weight the solution flows slowly through the wicking to keep the protruding end saturated. When the end of the wick is placed in a Bunsen flame, it furnishes a bright, long-lasting source whose second-order spectrum can easily be observed with a 14,000 line per inch replica grating. The observed spectra always contain the sodium *D* lines, but as yet no copper lines have been observed. Excellent results have been obtained using solutions of the chlorides of lithium, calcium, strontium, and barium. For obvious reasons it is necessary to have a different tube for each solution that is used. However, the simplicity and low cost of the device makes it feasible to provide each group of a large laboratory section with several tubes containing different samples.

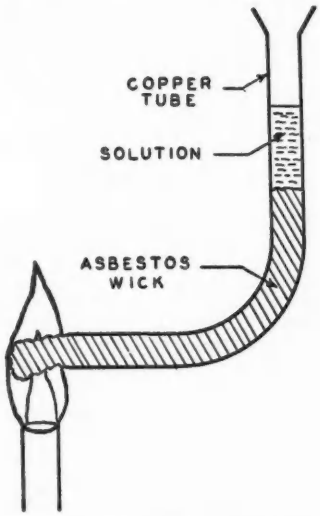

Fig. 1. Diagram of flame source.

Since the wicking must be packed tightly to retard flow, it is desirable to fill the tubes and allow them to stand for several hours before using.

Latent Heat of Vaporization Experiment

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THE error in finding the latent heat of vaporization by the conventional method using a calorimeter is often large, whether the experiment is done by the instructor as a demonstration, or by students. While experiments in the elementary course are not usually expected to give precise results, it is hard to "save face" when errors of 10 percent or more occur.

An alternate method of doing this experiment is to use a Liebig condenser and an ordinary distilling flask. The equipment is simple, the heat exchange relations are easily understood, and two or three determinations can be made during a two-hour laboratory period (see Table I).

The flow of cooling water from the tap into the condenser was adjusted so that its final temperature was about as

TABLE I. Results of several trials made by students.[a]

Trial	1	2	3	4
Temperature of water into jacket (°C)	15.0	15.0	15.0	15.0
Temperature of water out of jacket (°C)	32.0	32.0	35.5	37.0
Temperature of water condensed (°C)	23.0	25.0	21.0	21.0
Weight of steam condensed (grams)	25.0	25.0	24.7	25.4
Weight of cooling water (grams)	890	885	750	725
Heat of vaporization cal/g	530	527	544	550
Percent error (correct value 539 cal/g)	1.7	2.2	0.9	2.0
Percent error using calorimeter	10.0	7.9	7.6	62.0

[a] For comparison, the percent error is given for four trials using a calorimeter.

much above room temperature as its initial temperature was below. A constant-water-level tank may be needed to maintain steady flow of cooling water. The condenser was inclined at about 15 degrees to the horizontal. Several minutes were allowed for temperature equilibrium to be reached after all adjustments were made before readings were taken. Student grade thermometers were used; the two that measured the initial and final temperature of the cooling water were checked against each other. No lagging was used about any part of the system.

This is the method of Awbery and Griffiths[1] without their refinements. The results obtained are reported here because they are consistently better than might be expected from such simple procedure.

[1] J. H. Awbery and E. Griffiths, Proc. Phys. Soc. (London) **36**, 305 (1924).

A Demonstration Monocular

HOWARD N. MAXWELL
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FOR some time I have felt the need for a working model that would show clearly the construction and operation of the prism system of a binocular. With the ending of World War II and the consequent availability of optical parts, the necessary lenses and prisms were purchased at a nominal cost and arranged as follows:

In one end of a plywood box 6×10×24 cm is mounted the achromatic objective lens, which is five cm in diameter. The drawtube containing the elements of the eye lens is properly mounted in the opposite end of the box. The two prisms are rigidly attached to the bottom of the box and are so oriented that the whole optical system functions as a monocular. To keep out dirt, but still render the optical system completely visible to the students, a piece of plate glass has been cemented to and forms the top of the box.

In use, the monocular is passed around the class, so that each student has a chance to look through it at some object and also to observe the prism arrangement. It is not necessary to cover the glass plate with opaque material when looking through the apparatus, as the stray light entering through the top does not greatly interfere with the clarity of the image.

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LETTERS TO THE EDITOR

The Melde Experiment

IN the laboratory and particularly in the lecture room, it is sometimes desirable to have larger equipment than that usually available for the Melde experiment.

Figure 1 shows apparatus that is capable of setting up standing waves on a rope or heavy cord that will reach across the lecture room. Except for the motor and variable transformer, the apparatus may be constructed for little or no cost with materials usually found in the laboratory or shop.

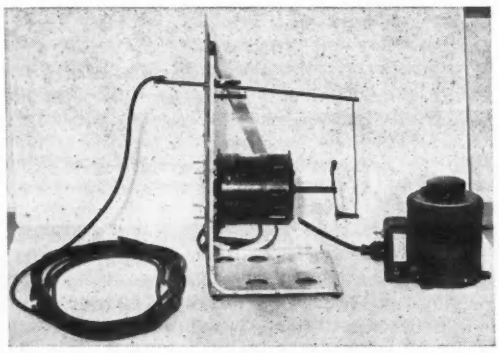

FIG. 1. Motor-driven device for generating standing waves in a rope.

The rope is attached to a rocker arm which is set into oscillation by the crank and rod which are in turn attached to the motor shaft. The speed of the motor is controlled by a Powerstat-type transformer. By varying the tension on the rope and the speed of the motor, standing waves of various length may be demonstrated.

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Concerning the Frequencies Resulting from Distortion

IT is a well-known fact that whenever a monochromatic train of waves is distorted in some fashion, additional frequencies appear. In the case of a half-wave rectifier, these additional frequencies are zero, two, four, etc., times the original frequency. For a full-wave rectifier, the frequencies are zero, two, four, etc., times the original, but the original is now absent.

For a wave train that has been so distorted that it satisfies the conditions $f(x)=0$, $-\infty \leq x \leq a$; $f(x)=\sin x$, $-a \leq x \leq a$; $f(x)=0$, $a \leq x \leq \infty$, Fourier's integral shows that a graph of the absolute values of amplitude, plotted

against frequency, has a form somewhat similar to that of a single slit diffraction pattern.

Other examples, such as continuous or damped wave telegraphy, and the unsymmetrical vibrations of the ear drum can be cited as instances where frequencies not originally present are introduced because of some type of distortion.

By use of either Fourier's series or integral, it is relatively simple to explain mathematically the afore-mentioned facts. What is the physical explanation of the origin of these additional frequencies?

HOWARD N. MAXWELL

Ohio Wesleyan University, Delaware, Ohio

The Swaying of a Catenary

IN the October 1950 issue of the *American Journal of Physics*, p. 416, I described experiments made on the swaying of a catenary. This catenary was formed by a chain of mass 77 g to the meter and of 57 links to the meter. Vibrations were taken perpendicular to the plane of the catenary and also in the same plane. I remarked that I did not think that the calculation of the period of a swaying catenary had ever been done. Prompted by this remark, Dr. E. U. Condon, then Director of the National Bureau of Standards, assigned this mathematical problem to David S. Saxon and A. S. Cahn of the National Bureau of Standards station at Los Angeles, California. Their solution has now been published under the title of "Modes of Vibration of a Suspended Chain" (Report No. 1309, 1951) and I have the liberty to quote from it. To be brief, I may say the report gives the mathematics and mechanics necessary for the solution, and tabulates the values of certain functions which enable a reader to work our readily for any given catenary the period of sway in its own plane, not only in the fundamental but also in the higher modes.

The catenary quoted in my paper had the following dimensions: whole length, 458.0 cm; whole span, 263.5 cm; sag, 169.7 cm. The constant c in the usual catenary equation is therefore 69.7 cm. The period of the simplest mode of vibration in its own plane was determined by me to be 1.80 sec. The period calculated by the use of Saxon and Cahn's tables works out to be 1.79 sec. The agreement is good.

It now remains for some mathematician to calculate the period of vibration of a catenary swaying forwards and backwards in a direction perpendicular to its vertical plane of rest. This solution will likely involve Bessel's functions, as does the problem of a swaying chain held by the upper end, which was worked out in the same October 1950 paper in the *American Journal of Physics* where I followed H. Lamb. Just as this chain does not remain straight, the catenary will not always be in a plane. In my paper, I assumed the catenary could be treated as a rigid body and I calculated the period of vibration of the above-quoted catenary in this direction to be 2.28 sec. Experiment gave 2.2 sec, a fair agreement.

I was pleased to think that my paper on some of the experiments we assign to our students had inspired the Bureau of Standards to tackle the problem of the swaying catenary.

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A Mercury Light Source for Use with a Diffraction Grating

AN ordinary fluorescent lamp provides not only an economical spectral source of mercury light, as pointed out by Kirkpatrick[1] and implied by McCay and Bishop,[2] Schwinn,[3] and Hilton,[4] but also furnishes a first-class substitute for the source slit itself in an ordinary grating experiment.

For this purpose the fluorescent lamp is mounted lengthwise on its metal base or reflector, and the mount gripped with a large size finger clamp in such a manner that the tube may be rotated concentrically about a supporting vertical axis or ring stand. The optimum extent of this rotation of the tube and its reflector backing from darkness to dawn is governed by sighting from a point near the grating when it is clamped before the eye at a distance of say a meter or more from the substitute slit. The width of the "slit" is therefore easily controlled, and equally as easily placed at the central zero mark on a lateral, double-meter stick.

The green and the yellow lines of the mercury spectrum may thus clearly be seen, and their lateral positions with respect to the tube, now acting as the slit, may easily be measured. These images stand out prominently enough to be located in spite of the unshielded western afternoon light of the laboratory, and notwithstanding the siren call of spectra arising extraneously from every plated doorknob and polished gadget, especially if a crossed grating is used.

Results with first-year students show a satisfactory approach to accepted values of the wavelengths for the prominent lines, it being sometimes possible to resolve and to reach the second-order spectrum with a fairly good replica grating.

One advantage of the long slit, attained from a conventional fluorescent lamp held vertically, is its extension above and below the horizontal meter scale. This juxtaposition makes for ease of reading. The slit logically may be set at the zero mark both for lateral and for normal scale starting points. For best results eye and slit should be on a be on a plane equally elevated on the table.

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[1] P. Kirkpatrick, Am. J. Phys. **15**, 359 (1947).
[2] M. S. McCay and E. S. Bishop, Am. J. Phys. **16**, 361 (1948).
[3] M. W. Schwinn, Am. J. Phys. **15**, 279 (1947).
[4] W. A. Hilton. Am. J. Phys. **19**, 248 (1951).

Focusing by Diffraction

THE recent study of transmission zone plates by Ora E. Myers, Jr.,[1] is interesting because it seems to fall in with the trend to return to neglected diffraction effects for focusing radiation. That good images can be formed by diffraction is known to any one who has looked at the many

colored images of a Geissler tube as seen through a diffraction grating. The zone plate, besides being able to form virtual images, can form real ones. Recently, Gabor[2] has shown that image formation is possible by using the complicated diffraction pattern formed by recording the shadow of an object illuminated by a point source on a photographic film. He calls this a "hologram." The object can be reconstructed by illuminating the hologram with light from a point source.

Rogers[3] has shown that Gabor's diffraction pattern behaves like a Fresnel zone plate in its focusing action. He shows that the diffraction pattern, produced when a small object (a point scatterer) is illuminated by a point source of monochromatic radiation, is well approximated by a set of concentric bands alternately light and dark whose boundaries have radii which are proportional to the square roots of the integers.

I have verified Rogers' equations and have produced reconstructions. In doing so, I have observed the circular pattern produced by a stray piece of dust. This leads me to suggest the following procedure for making a zone plate for x-rays. Form the approximate Fresnel zone-plate pattern by photographing the interference pattern of a small object. Minify (i.e., use magnification <1) this photographically on fine-grain Reprolith film of the peeling type. Peel the processed emulsion off and deposit it on an optical flat of beryllium. This should possess the proper transmission and diffraction properties for x-rays of the proper wavelength. This may be easier than making holograms with x-rays.

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[1] Am. J. Phys. **19**, 359 (1951).
[2] Proc. Roy. Soc. (London) **A197**, 454 (1949).
[3] Rogers, Nature, London **166**, 237 (1950).

Lecture-Room Measurement of the Value of g

FROM my experience in undergraduate level work I have found it very convincing to measure an important constant in the form of a lecture room demonstration. I have used for quite a number of years a physical pendulum to measure the value of g from free fall in the classroom.

The apparatus used, as it is shown on the diagram, consists of a thin rigid steel bar the length of which is 145 cm. The fulcrum (horizontal knife-edge support) is at the middle of the rod and a heavy adjustable mass (I used 5 kg) is fixed in the lower half of the pendulum so that the half period of the pendulum should be as close as possible to 0.500 sec. The apparatus is fixed on a heavy hardwood board which can be hung on the wall for use. The lower edge of the board is supplied with two screws for proper leveling of the knife-edge support.

TABLE I. Data on pendulum experiment.

	s in cm	50 T in sec
	122.58	50.2
	122.67	49.8
	122.71	50.6
	122.56	50.4
	122.72	50.3
Total:	613.19	251.3
Average:	122.64	50.26

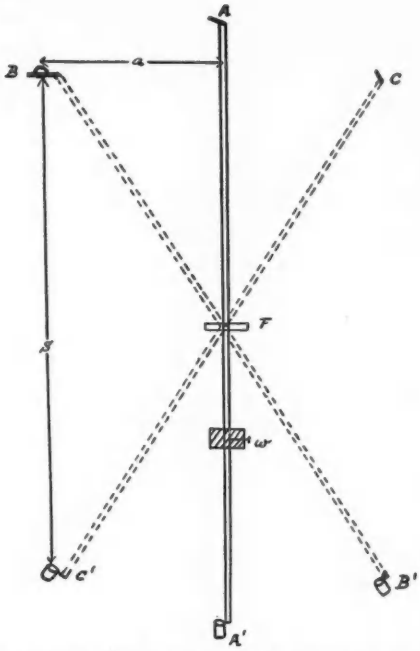

FIG. 1. Physical pendulum to measure the value of g from free fall: A–A' pendulum at rest. B–B' pendulum at start of motion. C–C' pendulum at the moment the steel ball is in mouth of the bag. a the perpendicular distance between center of washer and the center line of the pendulum. s the distance covered by the falling ball. f fulcrum. w weight.

One half of a washer (2-cm inside diameter, cut into halves along its diameter) is welded to the upper end of the bar and the second half in appropriate height is fixed to the board in such a way that the two halves should fit together when the pendulum is drawn aside. A little knit bag hanging from a ring the inside diameter of which is 3 cm is fixed to the lower end of the pendulum.

The pendulum held fixed by a thread in an extreme position where the two half-washers fit together is ready to use. A steel ball, 2.1 cm in diameter, is placed on top of the washer and by burning the thread the pendulum and the ball are released. If the timing of the pendulum is correct the ball falls into the bag.

The distance s between washer and the position of the mouth of the bag where the ball drops into it, in case of a bar 145 cm long, is $s=122.5$ cm, if the perpendicular distance a from the center of the washer to the center line of the pendulum rod in its rest position is 39 cm.

The distance s can be measured in the classroom by using a plumb line (I prefer this way), or by using a cathetometer. The period of the pendulum should be established by measuring the time of at least 50 periods, a stop watch used in the laboratories is accurate enough.

This experiment is a lecture room demonstration for students having their first course in physics. This is by no means an accurate laboratory method to measure g. However, it can be refined to such an extent that it could be used in the freshman laboratories. If the instrument is used in the laboratory the ring holding the knit bag should be replaced by a smaller one. I used with good success a ring having a diameter of 2.5 cm. The distance covered by the falling ball should be measured by telescope and the period of the pendulum has to be determined with the utmost care.

A few data in Table I taken from a lab report will show how it works.

From the above value $T/2=0.503$ sec, and hence, $g=969.5$ cm sec^{-2}.

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Gram *versus* Pound and Slug

THE question of "slugging a case for the pounders" or "pounding a case for the sluggers" is not merely one of scientific and engineering value. This question is of considerable pedagogical interest.

There arises in the first instance the problem of "what system of units should be used." It is, of course, appreciated and taken for granted that nobody can do away with the English system of units. (How much simpler a lesson could be, cries the pupil, if we had only the metric system!) But if we go as far as introducing a fourth dimension, namely F, could we not also make an effort towards that end with hope for a quick finish?!

I am unable to judge if the slug is found and used very much in English scientific or engineering literature. I have, therefore, to take for granted both J. G. Winans'[1] and S. L. Gerhard's[2] statements, which is not very easy. But it seems to me that had we used throughout the gram-mass and the gram-weight we should not have run into a continuous slugging. Let us use a gram-mass unit, kept as a standard and fundamental quantity. The multiple of it, the kilogram, does not present any difficulty. Then the gram-weight unit is defined as the weight of that standard mass, being a force unit. Nothing follows these definitions. No other unit of mass is henceforth introduced. As S. L. Gerhard says,[2] "The slug has failed." How does one understand the following definitions and explanations?[3]

"Accelerating force:

$$\mathbf{F}=\frac{W\mathbf{a}}{g}=M\mathbf{a}, \qquad (24)$$

where $\mathbf{F}$=accelerating force, lb, W=weight of the body, lb, $\mathbf{a}$=acceleration, ft per sec^2 produced by $\mathbf{F}$, and $g=32.2$ ft per sec^2.

"The quality W/g is commonly referred to as the mass of the body and is denoted by the symbol M. In engineering practice, length, time, and force are generally considered to be the fundamental quantities, and the system of units based on these quantities is called the gravational system. Mass, therefore, is a derived quantity. Use of this term, and of the concept of mass often simplifies discussions of bodies in motion. However, in the solution of practical engineering problems, the derived quantity M must always be replaced by the fundamental W/g, in which W represents force (pounds) and g length and time (feet per second per second).

"Equation (24) shows that the force required to give a body a definite acceleration a is proportional to its mass $(M=W/g)$."

Nowhere in the book quoted did I find the slug (I hope I did not overlook the poor beggar), but it seems as if implicitely that quotation is a definition of a unit, without giving it a name, which is worse still. Or is it S. L. Gerhard's $\mathbf{F}=m\mathbf{a}/g$ if one writes m for w and weight for mass?

It seems to me that we can get along quite well with three dimensions MLT, using the gram-mass, the gram-weight, and their multiples. The gram-mass, the dyne, and their multiples present the alternative system. The convenience to the greatest number of people certainly demands to choose the gram and not the pound or slug.

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[1] J. G. Winans, Am. J. Phys. **19**, 439 (1951).
[2] S. L. Gerhard, Am. J. Phys. **19**, 440 (1951).
[3] Alfred Jensen, *Applied Engineering Mechanics* (McGraw-Hill Book Company, Inc., New York, 1947), p. 175.

Cooling by Immersion in Liquid Air

WHEN objects at room temperature are immersed in liquid air some of the liquid air boils away. This boiling occurs at a regular rate for a considerable length of time. Finally, however, there is a tremendously increased rate of boiling which lasts briefly and then stops abruptly.

It is well known that when liquid air is poured onto a flat surface at room temperature it breaks up into droplets which assume the spheroidal state. If an insulating board is used and the liquid air is poured so that some remains in a large drop where first poured, other smaller drops moving away, the large quantity cools the board beneath it until it wets the surface. Then the large drop boils away before the smaller drops, showing that the rate of heat transfer is greater across the smaller temperature gradient existing between the board and the large drop.

If a few cubic centimeters of water are poured onto the surface of liquid air they will float partially submerged and may be observed through the liquid air from the side. The water freezes, resulting in an opaque blob of ice. Reflection of light from the surface of the liquid air under the ice shows that the surface is smooth, indicating the presence of a layer of air between the ice and the liquid air exactly as in the spheroidal condition. The wrapper of air is a very poor heat conductor, hence, the cooling of the ice is very much slower than would be the case if the liquid air were in contact with the ice. When the ice is nearly at the temperature of the liquid air, the wrapper of air may be seen to draw back across the bottom of the blob of ice; pursued by violent local boiling from points on the surface of the ice, the resultant bubbles propelling the ice around in the Dewar vessel. The thermal gradients which give rise to this last effect must necessarily be small, hence, the transi-

tion from spheroidal cooling to contact cooling occurs at a fairly sharply defined temperature. The contact cooling is very efficient and ends rapidly, the last few bubbles forming slowly from points on the bottom of the ice drop.

Thus the explanation of the phenomena outlined in the first paragraph is that spheroidal cooling first takes place and then changes to the more efficient contact cooling, giving rise to the final increased rate of boiling.

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The Contribution of the Joint Program of The Office of Naval Research and the Atomic Energy Commission to the Supply of Trained Scientific Workers

THE Office of Naval Research was permanently established by law in 1946.[1] Within the scope of the establishing legislation, the Nuclear Physics Branch of ONR initiated a broad program of fundamental research in nuclear physics. In 1948, in order to maintain this research at a productive level and at the same time to effect certain economies and simplifications in administration, the ONR and the AEC established the Joint Program of fundamental research in nuclear physics. The Joint Program currently comprises 42 active contracts. Five of these contracts are with industrial or foundation laboratories, and the remaining 37 are with 27 different universities.

Although the training of scientific personnel is not one of the specific purposes of the Joint Program, the accomplishment of such training is perhaps one of the most valuable by-products of this program. In view of the rather rather general postwar interest in the nation's technical manpower supply,[2–6] an attempt has been made to evaluate the Joint Program in terms of its contribution to the supply of trained scientific workers.

The Joint Program has made a substantial contribution to the graduate training of 27 percent of all Ph.D.'s in physics during the academic years 1946–47 through 1949–50. On the basis of some unpublished data,[7] the author has estimated that these men constitute at least 75 percent of all Ph.D.'s with specialization in nuclear physics trained during this same period. Thus, it would appear that in the process of sharing the financial burden of nuclear research the Joint Program has, at least tacitly, assumed a substantial responsibility to the graduate-training process throughout the nation, a fact which should be kept in mind by Joint Program officers and educators alike.

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[1] Public Law 588, 79th Congress, 2nd Session; Chap. 727 (H. R. 5911).
[2] Trytten, Sci. Monthly **60**, 37 (1945).
[3] V. Bush, *Science, The Endless Frontier* (OSRD, 1945).
[4] Steelman, *Science and Public Policy, A Report to the President* (U. S. Government Printing Office, 1947), Vol. 4.
[5] M. W. White, Rev. Sci. Instr. **18**, 219 (1947).
[6] M. W. White, Am. J. Phys. **19**, 27 (1951).
[7] American Council on Education Research Staff on Scientific Personnel, *Production of Scientific Doctorates: 1936–48* (unpublished).

Does High School Physics Help?

IN the first college physics class ever attended by the author, the teacher remarked that he hoped we had not taken high school physics. It appears that this attitude is not uncommon among college teachers.

Recently there arose an opportunity to compare the college physics work of students who did and who did not have high school physics. The group under investigation consisted of 876 students who had completed a two-semester course in college physics at Louisiana State University. Of this group, 367, or about 42 percent, had taken a year of physics in their high school programs.

For purposes of comparison, the letter marks were given numerical equivalents, with A=4, B=3, C=2, D=1, and F=0. The year mark for a particular student was arrived at by adding the two numerical equivalents for his semester marks. For example, two semester marks of A gave a student a year mark of eight.

The 367 students who had taken high school physics compiled a mean year mark in college physics of 4.856. Their standard deviation was 1.81. The 509 students without high school physics had a mean year mark in college physics of 4.758, with a standard deviation of 1.73.

On the basis of the standard error of the difference of two means, the difference between these groups was not significant. It was interesting to note, however, that in several similar studies, comparable slight differences were found. In practically all cases, these differences favored the group having high school physics.

So, although this comparison shows no outstanding difference in college physics achievement for the two groups, there seems to be no support for the position that a course in high school physics is a handicap in college physics.

Meanwhile the slugfest goes merrily along. The college teachers say that the high schools have left subject matter out of their curricula. But the high school teachers retort that college teachers know nothing about how to teach, and anyhow, high schools are no longer supposed to be college-preparatory.

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The Complete Physics Course through Electric Trains

WHEN the elementary physics class at Brooklyn College assembles for the first laboratory period, in addition to the usual micrometers and calipers, there is found a small *HO*- or *TT*-gauge track layout with an engine and a few cars. Complete *HO* and *TT* outfits or track layouts with rolling stock, complete with scenery, can be purchased for about $140 and *O* gauge sets in proportion. A cheap train set complete with transformer may be purchased for about $10, or less. If the entire sum is not available the layout can be built from parts, as the course proceeds, for a fraction of the cost of a finished layout. An *O*-gauge track layout for trolley cars can be compressed

into a 30-inch circle and a good working model of a trolley car may be bought for about $20.

No window display exceeds electric trains and trolleys for eye-catching appeal and this class is no exception. Students are all eager to measure the track gauge in centimeters and inches, the mass in grams and pounds, and to convert the gallon capacity stamped on the tender into liters. Before they know it they have mastered the three fundamental relations; 2.54 centimeters equal one inch; 2.2 pounds equal one kilogram; and 231 cubic inches equal one gallon. A few sections of track can be bent into circles to illustrate radian measure and curvature of arc.

The kinematical equations of motion may be described as the train runs forward and backward with acceleration and at constant speed. One of the freight cars has marble or steel balls in it; they may be dropped on the floor to illustrate the acceleration of gravity and later used for demonstrating the coefficient of restitution.

While the train runs, a marble may be pushed sideways from a flat car to illustrate the composition of vectors and in this way the entire class can see the actual motion of the projected marble. The trains have smoke units in them and by means of electric fans the smoke can be given different directions. The trains can also be made to pass each other on parallel tracks to illustrate relative velocity problems. If the track is oriented northeast the velocity of the train can be resolved into components. A small balloon tied to the locomotive while it is in motion will furnish the basis for wind problems.

A piece of loose track may be used as a seesaw by being put over a knife edge and this shows that two cars at one-half a particular distance balance one car at the full distance. Small ladders can be placed against the sides of the cars as a basis for the study of the ladder and moment problems.

When the train is set in motion, a discussion of the force necessary to pull the cars leads to the F equals ma relationship and the time required to set a heavy train in motion leads to the impulse equation. Friction can be introduced as a topic at any point here.

A spring balance is hooked onto the locomotive and it is raised and lowered with an acceleration. The locomotive is weighed and then hung on one end of a cord passing over a pulley with a suitable counterweight on the other end of the string for an Atwood-machine demonstration.

A few sections of track are laid on an incline and the locomotive is run up and down the incline to illustrate motion up an incline with friction and acceleration. The locomotive on a straight piece of track can be pulled along by a weight attached to the locomotive by a string passing over a pulley at the edge of the table; or it can be pulled up the incline by the same device. As the train moves along a small spring shoots a marble from one of the gondola cars straight up into the air and it falls back into the car.

The gravitation law may be illustrated by any two objects and the locomotives are no exception. The draw bar pull of the locomotive can be measured and this leads to a discussion of work, energy, and power. Kinetic energy is well illustrated by the motion of the train. The track as an incline illustrates a simple machine, and there are plenty of wheels, axles, and gears to deal with. The locomotive can be lifted by a small differential pulley and a wedge may be used to keep the track from slipping or a car from rolling down hill.

On the curved portion of the track, if the locomotive runs too fast there is no doubt in the student's mind as to which set of wheels, the inside or the outside ones, leave the track. This observation leads to the formula for a banked track, and a piece of track in the form of a vertical circle with a small car leads to looping the loop. The train can be run at constant speed in a circle and a board put up along a diameter to catch the shadow to illustrate simple harmonic motion. The rotating wheels illustrate the lecture on rotational kinematics and the torque in the motor and driving wheels leads to the L equals $I\alpha$ relationship.

Fastening the cars together with wire leads to the elasticity concept, and the reaction of the track leads to the shear modulus. Cotton or Kleenex may be compressed into bales and used to demonstrate bulk modulus. Pressure in the bottom of the tank cars takes the class into hydrostatics, and putting the whole train on a barge will serve as a good illustration of Archimedes' principle.

This analysis could be kept up *ad infinitum*, and it seems to the author that every experimental topic in physics can be clarified and made more interesting by the use of the electric train. In fact, a great many schools have always used a horizontal circle of track free to rotate and a clockwork or electric engine to illustrate the third law of motion and the conservation of momentum. The demonstration is used as a Christmas window display by many toy stores. With a complete track layout the whole elementary physics course might take on new life and meaning for the average student.

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Meanwhile machines deprive us of two things which are certainly important ingredients of human happiness, namely, spontaneity and variety. Machines have their own pace, and their own insistent demands: a man who has an expensive plant must keep it working. The great trouble with the machine, from the point of view of the emotions, is its regularity. And, of course, conversely, the great objection to the emotions, from the point of view of the machine, is their irregularity.—BERTRAND RUSSELL, Sceptical Essays (1928).

ANNOUNCEMENTS AND NEWS

Book Reviews

The Earth's Magnetism. Second Edition. SYDNEY CHAPMAN. Pp. 127+xi, Figs. 39, 10.5×17 cm. Methuen and Company, Ltd., London, John Wiley and Sons, Inc., New York, 1951. Price $1.50.

Although the jacket of this book says it is intended for those with some familiarity with the subject, it is also easily readable for others. Besides those reading for general interest, those planning a wider study of the subject will find it a good introduction, especially to the two-volume work co-authored with Bartels.[1] Much material has been condensed here, and one must read elsewhere for exposition of some details. For example, in the discussion of magnetic potentials where first geomagnetic coordinates and then geographic coordinates are used, the large books must be consulted for the lengthy explanation.

The book is essentially descriptive of the earth's field and its comparatively short time variations. The main field is taken up in about one-fourth of the book, while the remainder deals with field variations due to the sun and moon and attendant latitude and longitude relations. Upper air currents are treated briefly but effectively. The long time, or secular variation, causes for it and for the main field, instruments, and geomagnetic prospecting, are discussed very little. The speculations regarding particle radiations from the sun are very intriguing.

It is of little inconvenience in this small book, but there is a practice common to many authors of referring in early chapters to sections much later in the book for clarifications. Better labeling of figures would be helpful, and one would appreciate being oriented on a figure before rather than after discussion of it.

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[1] Sydney Chapman and Julius Bartels, *Geomagnetism* (Oxford University Press, London, 1940), first edition. Second edition said to be in preparation.

Panorama of Science, 1951. Compiled and edited by WEBSTER PRENTISS TRUE, formerly Chief of the Editorial Division of the Smithsonian Institution. Pp. 416, Figs. and plates 115. Series Publishers, Inc., New York, 1951. Price $5.50.

This is the 1951 supplement to the original twelve-volume Smithsonian Scientific Series (1929), Volume I of which was written by Mr. True. Unlike the detailed descriptions of the multifaceted Smithsonian scientific and cultural exhibits, researches, and expeditions contained in original series and the annual supplements, however, this volume is a selection of outstanding articles which appeared in scientific journals during 1949 and 1950. Many of the articles are taken from journals well-known to physicists such as *Physics Today*, *American Journal of Physics*, *Sky and Telescope*, and *The American Scientist*. Other journals represented are perhaps less familiar, e.g., the *Bulletin of the American Association of Petroleum Geologists*, *The Wilson Bulletin*, *Journal of the Institute of Navigation* (*British*), and *The Auk*.

Mr. True has set aside his facile pen to let the scientists speak for themselves. His comment on this is that "any attempt to reshape their findings into sugar-coated condensations is sure to result in loss of knife-edge accuracy and of something of the feel of the scientists' own thinking." Although each of the authors is an authority in his own field, the articles represent various levels of not too technical discussions which appeal to an audience such as that reached by *Scientific American*. There is, however, a subtle but distinct difference between the articles in *Panorama* and those in *Scientific American* in that the former are written by scientists for scientists rather than for laymen. Generally speaking, the moderately well-read scientist in any particular field will find the articles from neighboring and distant fields more interesting than the articles from his own field. The general lay reader interested in thoughtful and stimulating excursions to the contemporary frontiers of science will be well rewarded for his reading time.

Twelve of the twenty-seven selections are from the overlapping realms of physics and astronomy. Included are Otto Struve's lucid discussion of a new theory concerning "The Origin of Comets," Willy Ley's enthusiastic plans for "The Satellite Rocket," and E. C. Payne's intriguing description of "Electroluminescence—A New Method of Producing Light." Outside of this nucleus of the micro- and macrocosm are the remaining fifteen articles, no other field claiming more than two of them. Mr. True's selection rules have yielded several article-pairs in the fields of ornithology, conservation of natural resources, archaeology. ethnology, and the philosophy of science. The remaining articles include diversified topics such as "Maté—South American or Paraguay Tea" by R. Howard Porter and "The Promise of Geriatrics" by Thomas C. Desmond. Two articles from the *General Electric Review*, "Production of Enriched Nuclear Fuel" by K. H. Kingdon and "The Technical and Economic Significance of Developments and Trends in National Security" by several G. E. authors, do not appear to be sufficiently informative to merit inclusion in *Panorama*. Future editions of *Panorama* will be of more value and interest to both the physicist and layman if Mr. True's exclusion principle is relaxed to include other scientific areas, especially the life sciences.

In "Chinese Mirrors and Chinese Civilization" by Schuyler Cammann, a study of the workmanship and symbolism entailed in the decorative designs on the backs of Chinese bronze mirrors fabricated at various times from the first century A.D. to the twentieth century shows a gradual deterioration which parallels the decline of Chinese culture during the same period. The very excellent plates accompanying the article reveal clearly the centurywise degeneration of originally beautiful and intricate symbolisms into crude good luck charms which are now used

in the Chinese hinterlands. It is an ironic twist of history that this study was initiated by recent excavating processes involved in the building of roads and airstrips.

The philosophical problem of "The Limits of Science" is treated in a speculatory thought-provoking essay by Eugene P. Wigner, originally published in the *Proceedings of the American Philosophical Society*. The possibility of a not too far distant shift of scientific interest from physics to other fields, particularly the life sciences and psychology, is pointed out in realistic terms. However, the hypothesis that condensation of knowledge in physics has reached its limit of usefulness will most assuredly not be accepted unquestioningly by all physicists. In suggesting group research as a possible means of extending the limits of scientific knowledge "by the juxtaposition of several individual intellects," Dr. Wigner cautions that "the problem of group research is to avoid suppressing the subconscious thinking of the individual but to make available for him the information and to some degree even the unfinished ideas of his collaborators." Unfinished ideas have been a primary source of scientific advance for three hundred years. Since the only fundamental difference now is in the mode of communication of these ideas, this reviewer poses the question as to whether the symbiotic association of scientists engaged in group research may not eventually provide a much more effective communication of ideas, finished as well as unfinished, than has ever been possible through letters and publications.

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College Physics. ROBERT L. WEBER, MARSH W. WHITE, AND KENNETH V. MANNING. Second edition. Pp. 820+viii. McGraw-Hill Book Company, Inc., New York, 1952. Price $6.50.

This book constitutes a second edition of the text, *College Technical Physics*, the book we associate with sketches of Nobel Prize winners. In the words of the authors, this book is designed "for students of engineering and science." It is well adapted for use at this level; it is probably too long and refers to too many separate concepts (the index lists 1700 items) for successful use in an arts-college course in physics.

The organization of the book is conventional. The subject matter is presented in the traditional order: Mechanics, Heat, Sound, Electricity, Light, and Modern Physics. Anyone who has taught introductory physics will experience little trouble in adapting himself to this text. Things are in their usual place.

The exposition is clear and orderly. Each new term appears in italics and is accompanied by a definition frequently also in italics. Many solved problems are inserted into the expository portion of the book whose purpose is that of illustrating the application of the material being presented.

This reviewer cannot resist at this point raising one objection which may not seem significant to many instructors. Should not a section appear in a book on this level whose sole purpose is that of defining a definition? We know that many of our definitions are operational definitions; we know that students generally only understand definitions like "A horse is an animal with four legs"; we know that basic element of the logic of physics has been left behind if the role of operational definitions is not discussed. Yet this book is not alone in this neglect.

At the end of each chapter appears a summary, a list of questions, and a list of problems. The summary has as its purpose the direction of the student's attention to the basic principles that have been presented. Unfortunately, the requirement of brevity has forced the authors into an occasional corruption of these principles. Statements like "*Polarization* is the process by which vibrations of a wave motion are confined to a definite pattern"; "A body is in *equilibrium* when there is no change in its motion"; "For every acting force there is a reacting force equal in magnitude but opposite in direction"; and "Waves are *diffracted* by obstacles in the medium" are likely to be a bit disturbing at first. Actually this feature of the summaries is undoubtedly inevitable. It would indeed be an inert student who would expect to develop a well-rounded grasp of physics by reading only the summary sections in any textbook!

The authors have really gone all the way on questions and problems. By actual count the book has 1524 problems and 820 questions. Pairs of similar problems are provided, the answers being given to only one of each pair. The questions are very intriguing. Do *you* know the scientific basis for the weather proverb, "*The number of stars within the moon's halo denotes the number of days until rain*"?

JOSEPH W. STRALEY
University of North Carolina

Philosophical Problems of Mathematics. DR. BRUNO BARON V. FREYTAG GEN. LÖRINGHOFF. Translated from the German by Amethe Countess von Zeppelin. Pp. 88. Philosophical Library, New York, 1951. Price $2.75.

Both the histories of philosophy and of mathematics illustrate the irreconcilable conflicts as to which originated first and as to which has had the greater influence upon the other. Both also have had their claimants as to whether the origins have been of an earthly or of a divine ancestry. Galois probably knew if he believed the last letter he received from his father who wrote: "But even mathematics, the most noble and abstract of all the sciences, has its crown in the air, but its roots deep in the earth on which we live." Not only its roots, but its branches, even though mathematics has had always to go beyond mere experience into the realms of abstraction. And the roots are in the long forgotten past, because all mathematical concepts are the products of long lines of intellectual development. There has always been a dichotomous separation of the paths taken, one road remaining empirical-practical, the other, the royal abstract path to pure thinking. As Mill observed, "the first premises of mathematics from which the remaining truths are deduced are, notwithstanding all appearances to the contrary, results of observation and experience—founded in short, on the evidence of the

senses." As oversimplified as this observation is, it points up how the intuitionist-rationalist and the empiricist approach have been at such great odds.

E. T. Bell and W. W. Sawyer and others have shown how fallacious the "mysterious faculty" and "intuition" pleaders are. Most historians recognize how and when "pure" mathematics influenced the applied mathematics and *vice versa*, but not Baron v. Freytag, for he separates them. Kant's influence was tremendous in his belief concerning the *a priori* character of our geometric notions, but it did not move Gauss at all, a fact which v. Freytag ignores. For the Baron, mathematics is a matter of thought alone, agreeing with the leading intuitionist, Brouwer, and with Schlick who believes in the subjectivity of space and time, all of which are merely concessions to formalism and idealism.

Dr. v. Freytag, Professor at Tübingen, assays a philosophical approach to the science of mathematics. The Baron ranges over all mathematical and philosophical territory; he is successful to the degree only that he interprets his subject matter almost purely in neo-Kantian subjectivist terms. The philosophical interpretations are akin also to phenomenological ontology, and he speaks admiringly of the positivistic humanism of Hartmann and Jacobi. His beliefs concerning the mathematical sciences are that probability and logic will undergo transformation as they attempt solutions of philosophical problems. His views are constructive when he agrees with scientific philosophers such as Reichenbach that all sciences are intimately related to philosophy and, further, that scientists and philosophers need to get together and supplement each other's productivity. The breach between mathematics and philosophy will widen, Dr. v. Freytag believes, since the scientists are getting further away from the Idea of the Whole. Mathematics ought not to be regarded as philosophy, nor *vice versa*, since there is a great deal more of philosophy in such mathematical disciplines as the Theory of Sets and the Theory of Proof. Both mathematics and philosophy, Dr. v. Freytag alleges, are preoccupied with the unsolvable problems of Infinity. Not even Goedel's theorem on systems will solve Infinity problems.

Dr. v. Freytag's main concern is his brief for the mathematical philosophy of the rationalists Pythagoras, Plato, Cusanus, Descartes, and Leibniz, forgetting that the latter regarded mathematical truths only as necessary to serve as the universal exposition of possible forms of connection in general and their mutual dependency. Baron v. Freytag talks about the objects of mathematics meaninglessly, for what he does mean are "pure ideas" dealt with by the pure science, mathematics. Or, he means Being and the problem of the applicability of mathematics to concrete reality (Being), never real at all, but "thought relations between an abstract entity and the ego," which is "the independent Subsistence in itself"—"ontological essence."

The ideas on space and time, Space-Time, spatio-temporality, space-time quality are idealist to the extreme, even more so than Kant, whom he incorrectly regards as his mentor. He terms his work an aspect of *Deutsche Mathematik*, which is another indication of a mystical chauvinism, especially his belief that physically real data are only fictions. There is discussion of "genidentity," transcendent reality being unequivocal, the *a priori* character of natural knowledge, vast realms of Ideal Being, and the existence of a Divine Mind beyond the human mind. The best this reviewer can say is that this is at least about philosophy and mathematics.

EUGENE C. HOLMES
Howard University

Phase Microscopy Principles and Applications. ALVA H. BENNET, HELEN JUPNIK, HAROLD OSTERBERG, OSCAR W. RICHARDS. Pp. 320+xiii. John Wiley and Sons, Inc., New York, 1951. Price $7.50.

All of the authors of *Phase Microscopy* are associated with the Stamford Research Laboratory of the American Optical Company. A glance at the bibliography indicates that they have collaborated on the subject since 1946. This may explain the absence of serious discontinuities which might easily attend a book written by four authors each writing chapters independently.

Phase Microscopy claims to be the first book to present a working knowledge of the method. It includes a bibliography of over 350 published contributions which attest to the vigorous activity which has attended the subject since Zernike wrote the first paper in 1935.

The first chapter contains the following general statements about vision and microscopy: Almost all our knowledge of observed scientific phenomena is based on our ability to see. Yet the eye can recognize only differences in brightness and in color. The eye is unaware of phase, but if a microscope can delineate change of phase as a change in brightness or color, the eye will be able to detect the microscopic areas causing the phase change. More specifically, the role of the phase microscope is stated on p. 166: "The phase microscope utilizes absorption and optical path differences for the control of the contrast visible in the image. Phase microscopy is most valuable for specimens too transparent for examination with other types of microscopy. When the specimen detail consists of small differences in optical path due to variations in refractive index and thickness, regions of absorption, or both, the phase microscope should be used."

Pedagogically, this book is attractive because it presents the theory three times in increasing steps of sophistication. In the first chapter, Alva H. Bennett presents an explanation of phase microscopy which can be understood by a serious sophomore. Then all of Chapter II is devoted to "An elementary theory of phase microscopy." This can be read with profit by any student of optics because

(a) it discusses phenomena of all microscopes—in fact, the ordinary microscope is considered as a special case of a phase microscope; and

(b) it uses complex numbers and exponential notation to represent phase and amplitude. The simplifications introduced by this notation have been slow to enter American textbooks, but anyone who has derived the formula for a grating with N slits by both exponential and trigonometric functions knows the advantages of the former.

Harold Osterberg then opens all stops and has a mathe-

matical holiday with the theory for the third and last time in an Appendix, Chapter VII, entitled "The diffraction theory of phase microscopy with Köhler illumination." The reader will be innocently misled if he thinks he can jump from a nodding acquaintance with Kirchoff's formula to an understanding of this chapter.

Osterburg says, "In deriving the diffraction integrals governing image formation, we shall apply Kirchoff's laws as they have been reformulated by Luneburg (1944)." The nonmathematical reader will not get far in this chapter. The reviewer, who does not consider himself mathematically illiterate, was not able to follow the details of this chapter, possibly because Luneberg's article was not available. The conditions for the reduction of the general theory to the elementary theory of Chapter II are stated, and the properties of diffraction plates are given in terms of real parameters. One gets the impression that the treatment is thorough, but it would take time and references to check its details.

The great family of phase microscope users are neither physicists nor mathematicians. They will find Chapter IV on "Phase microscopy in biology and medicine," and Chapter V on "The technics of phase microscopy," and Chapter VI on "Industrial applications of phase microscopy" amply rewarding. These chapters are rich in illustrations and in tabulated data of use to microscopists, such as Table IV. 1, "Media for mounting specimens for microscopical examination" with 66 entries, and Table V. 1, listing 59 biological and industrial mounting materials and their indices of refraction. Applications to micro-organisms, cytology, killing and fixation, embryology, histology, and tissue culture will interest the biologist.

Industrial applications provide an impressive list including chemicals, crystals and minerals, clay and dust, foods, drugs and pharmaceuticals, fat, grease and soap, paints and pigments, glass and plastics, paper, metal surfaces, rubber, and textiles. The photographic illustrations are clear and instructive.

This reviewer was particularly interested in the possible use of phase microscopy for studying polished glass surfaces. "Since there is no satisfactory explanation of the nature of polishing of glass surfaces, phase microscopy promises information of both heuristic and practical value" (p. 230). Another form of microscopy, *viz.*, by x-rays, would be greatly aided by any knowledge of how to control the smoothness of glass surfaces. In fact, Zernike's paper entitled "Diffraction theory of the knife edge test and its improved form of the phase contrast method," which preceded all the work on phase microscopy, has rekindled interest in the optical testing of x-ray reflectors.

There is an extensive chapter (III) on instrumentation by Helen Jupnik. Here the functions and properties of the various elements of the microscope and a description of the necessary alignments and adjustment are given in detail. Photographs and descriptions of ten different domestic and foreign commercial phase microscopes are included. Excellent photographs illustrate the degrees of correction possible with different diffraction plates.

One might object to an apparent discrepancy in first defining optical path as the product of a length by refractive index (p. 17), thus making its physical dimension length, and later speaking of $\Delta=2\pi d/\lambda$, a dimensionless number, as an optical path difference (p. 19). This loose way of speaking may be forgiven among the initiates in much the same way as "pressure in mm of Hg" is permissible slang elsewhere.

Many physicists, biologists, industrial and research workers, and students are going to find *Phase Microscopy* invaluable. It is far from being a mere handbook for the use of phase microscopy. The emphasis on theory is well put, since subtle future uses will suggest themselves to those who understand the physical possibilities and limitations of the method.

ALBERT V. BAEZ*
UNESCO Technical Assistance Mission
University College, Baghdad, Iraq

* On leave from the University of Redlands, Redlands, California.

Report of the Editor for the Year 1951

The 1951 *American Journal of Physics* containing 580 pages was almost identical in length with its predecessor. In the Regular Section 84 articles appeared, averaging 5.27 pages in length. Under Notes and Discussion there were 35 items, averaging a trifle under one page. Sixty-one Letters to the Editor were included, usually about 0.5 page in length. Contributions of all kinds from outside the boundaries of the United States numbered only 14, representing no significant change from 1950.

In the service departments of the *Journal*, book reviews were printed frequently. A total of 50, occupying 31½ pages appeared.

At the date of the last report from the publication office, an edition of 5900 copies of the *Journal* was printed each month. This number allowed for distribution to 3107 members of the Association and to 1488 subscribers who were not members of the Association, and left something in reserve for back number sales.

The most important change in policy in the editorial office of the *Journal* was that beginning in January, 1951, the *Journal* followed the official style of the other Institute journals, although it has not yet adopted the more economical format to which its sister journals were forced by increasing publication costs. Details of the stylistic characteristics can be found in the Institute's Style Manual, a copy of which should be available in almost any department of physics. No other major change of policy was introduced in 1951.

Dr. B. H. Dickinson, the assistant editor, has supervised the analysis of the 1942 and 1951 volumes of the *American Journal of Physics* with respect to numbers of contributors, lengths of articles, and numbers of articles. The results of the analysis are presented in Figs. 1, 2, 3, and 4. Striking differences are noticed in the forms of the plots near the origin. The items A, B, C, and D below, state in words and figures what Figs. 1, 2, 3, and 4 express graphically.

A. In 1942 there were 13 contributions of about 0.1 page; 24, about 0.2; 12, about 0.3; 8, about 0.4; 5, about 0.5. Total, 62.

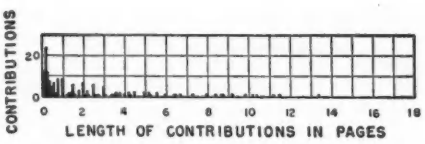

FIG. 1. Contributions to the *American Journal of Physics*, Vol. 10, 1942, separated according to lengths of individual items.

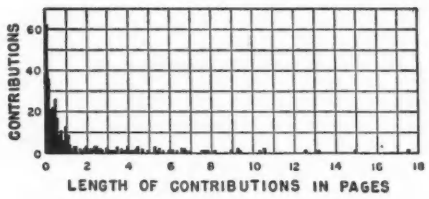

FIG. 2. Contributions to the *American Journal of Physics*, Vol. 19, 1951, separated according to lengths of individual items.

In 1951 there were 63 contributions of about 0.1 page; 36, about 0.2; 21, about 0.3; 22, about 0.4; 26, about 0.5. Total, 168.

Activities in physics were, of course, much more widespread in 1951 than in 1942. Volume 19, 1951, had 73 percent more pages than Volume 10, 1942. Even allowing for this increase, the statements made under (A) above reflect the *Journal's* present-day policy of recording as fully as possible the activities of the Association's sections, including abstracts of many papers presented orally at section meetings. Although the editorial work involved in handling brief contributions is, relatively speaking, considerable, the rewards to the *Journal* are great, since many papers accepted for publication have their origin in abstracts.

B. In 1942 there were 77 contributions of length 1.0 to 13.5 pages.

In 1951 there were 105 contributions of length 1.0 to 17.5 pages.

This change is of the order of magnitude that might have been expected on the basis of the total numbers of pages published in each of the years mentioned.

C. Many authors, of course, contribute more than one item to a volume. In 1942, 4 authors contributed a total of 0.1 page each; 20, 0.2 page each; 5, 0.3 page each; 6, 0.4 page each; 3, 0.5 page each. Total, 38.

In 1951, 40 authors contributed 0.1 page each; 26, 0.2 page each; 13, 0.3 page each; 14, 0.4 page each; 19, 0.5 page each. Total, 112.

Here the same change is apparent that was noted under (A), namely, that the *Journal* is recording minor contributions with an increasing frequency. This trend is part of a conscious attempt on the part of the editor to encourage wider participation in the *Journal* and to give encouragement to members who hesitate to submit manuscripts unless they be of considerable length.

D. In 1942, 80 authors each contributed 0.6 page or more to the *Journal*. Assuming that all authors were members of the Association, this figure represents about 8.4 percent of the membership.

In 1951, 145 authors each contributed 0.6 page or more. Assuming that all authors were members, this figure represents about 4.7 percent of the membership.

Here the lower limit of article length was chosen at 0.6 page to make sure that abstracts would not be counted. It is found that as the membership has grown from 950 in 1942 to 3100 in 1951, a significant falling off has occurred in the proportion of members who take an active, contributing part in *Journal* matters, from 8.4 percent to 4.7 percent. These figures are commended to the Council and to its Membership Committee for continued study.

Three Associate Editors concluded their terms of office on December 31, 1951. They were J. E. Goldman, H. N. Otis, and Eric Rodgers. These gentlemen gave freely of their advice to the editor in deciding which of the manuscripts submitted for publication should be accepted and which rejected. The membership at large is, I am sure, unaware of how many helpful comments made by our Associate Editors were ultimately incorporated in published manuscripts. The Association is greatly indebted to its Associate Editors and to a large group of competent physicists throughout the United States, who have been gracious enough to advise the editor on matters that were beyond the range of his specialty.

In place of the three retiring editors, the following men are recommended to the Council for appointment as Associate Editors for the three-year term, 1952–54. Ira M. Freeman, Rutgers University; A. C. Helmholz, University of California, Berkeley; C. N. Wall, University of Minnesota.

Many complimentary copies of new books were received in the Editor's office direct from the publishers. When one

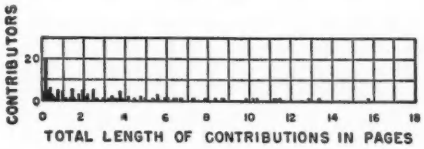

FIG. 3. Contributions by individuals to the *American Journal of Physics*, Vol. 10, 1942, separated according to the total length of material published under the name of an individual. In cases of papers by several authors, the first-named was credited with the item for the purpose of arriving at a total length.

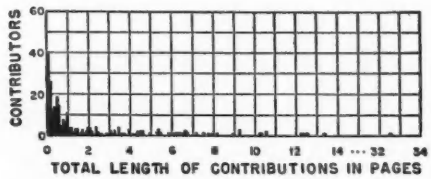

FIG. 4. Contributions by individuals to the *American Journal of Physics*, Vol. 19, 1951, separated according to the length of material published under the name of an individual. In cases of papers by several authors, the first-named was credited with the item for the purpose of arriving at a total length.

of these was appropriate for review in the *Journal*, it was sent to and remained the property of the reviewer. Some 25 volumes of a technical character, dealing with subject matter not strictly in the field of physical science or physics teaching, were sent as a donation from the Association to the Free University of Berlin at the request of the National Conference of Christians and Jews, Inc.

A great deal of the responsibility for the preparation of manuscripts in their final form and for the detailed editing has fallen upon Dr. B. H. Dickinson. Were it not for his conscientious work, many contributors would have been faced with the task of making much more extensive changes in proof than actually occur. The Association is very fortunate in its assistant editor.

It is a pleasure for the editor to express his continuing debt to the staff of the publication office at the American Institute of Physics headquarters, and to thank the officers of the American Association of Physics Teachers for their kindly counsel and sympathetic consideration of difficult problems that were presented to them.

THOMAS H. OSGOOD, *Editor*

Report of the Treasurer

December 31, 1951

Balance brought forward from December 31, 1950		$ 7,171.92
INCOME		
Dues received from AIP	$17,230.61	
Refund from AIP for excess of remittances over estimated costs	452.70	
Royalties	243.90	
U. S. treasury bond interest	300.00	
Check to cover American Council on Education dues	100.00	
Total income		$18,327.21
Total cash available		$25,499.13
DISBURSEMENTS		
Payments to American Institute of Physics		
American Journal of Physics publication	$ 7,560.90	
Collection of dues and miscellaneous services	1,595.53	
13 percent of dues collected in 1950 for AIP support	2,151.57	
President's office	86.77	
Secretary's office	854.46	
Treasurer's office, postage	45.95	
Editor's office	1,338.05	
Assistant editor	800.00	
Membership committee	573.73	
Purchase of U. S. notes	5,024.79	
Publication of AAPT directory	1,547.69	
American Council on Education dues	100.00	
Representatives to ACE meetings (H. K. Schilling; B. B. Watson)	151.12	
Ritchmyer lecture honorarium	100.00	
Engraving of 10 Oersted award certificates	100.00	
Expenses connected with AAPT meetings	27.69	
Total disbursements		$22,058.25
Balance on hand December 31, 1951		$ 3,440.88
The Association held on December 31, 1951, U. S. Government Treasury Bonds and Notes having a par value of		$15,000.00

PAUL E. KLOPSTEG, *Treasurer*

January 28, 1952

I have audited the books of account and records of Dr. Paul E. Klopsteg, Treasurer of the American Association of Physics Teachers, for the year ended December 31, 1951, and hereby certify that the foregoing statement of receipts and disbursements correctly reflects the information contained in the books of account. Receipts during the year were satisfactorily reconciled with deposits as shown on the bank statements, and all disbursements have been satisfactorily supported by vouchers or other documentary evidence. U. S. Government Bonds and Notes of $15,000 par value are held in safekeeping by the State Bank and Trust Company of Evanston, Illinois, and a certificate was obtained from the custodian as of December 31, 1951.

WILLIAM J. LUBY
Certified Public Accountant

Chicago, Illinois
January 25, 1952

As young investigators acquiring the principles and attitudes as well as the techniques of research, do they [graduate students] learn the rights of independent thought and action or the ways of suspicion and restraint? When they are introduced to research under conditions of secrecy and quarantine from fellow students and unauthorized faculty members, what is their initial and perhaps lasting attitude toward freedom of choice and expression? Where is the vaunted freedom of science, heard but not practiced?—N. PAUL HUSDON, Ohio State University Graduate School Record, December, 1951.

RECENT MEETINGS

Oregon Section

The fifty-eighth meeting of the Oregon Section of the Association was held at the University of Washington, Seattle, on November 3, 1951. The program began at 9:00 A.M. with papers as follows:

Welcome. DR. JOHN H. MANLEY, *Executive Officer, Department of Physics, University of Washington.*

Relation of crystal structure to field emission. W. W. DOLAN, *Linfield College.*—The coordination of fluorescent screen patterns obtained in field emission with crystallographic analysis of the single-crystal tungsten emitter presents problems that are important to both practical and theoretical aspects of field emission. Existing theories do not provide an adequate mechanism for the directional variation of work function, but the researches of Miller, Nichols, Smoluchowski, Stranski, and others contribute points of view that promise to lead toward eventual explanation of the phenomenon.

Field emission; the distribution of current density and its effects on the comparison with theory. J. K. TROLAN AND WALTER P. DYKE, *Linfield College.*—The wave-mechanical field emission law has been checked by experiments that measure simultaneously the emitted current I and the applied voltage V, and monitors the emission pattern. Emission pattern analysis indicates an area of essentially constant field at the emitter apex. Evaluation of the percentage of total current I emitted by the constant field region and the percentage of total emission over the area contained in this region affords an improvement in the evaluation of $\bar{J}$, the average J over the constant field region. This improves previously determined $\bar{J}$'s by a factor of 2 to 3.

Rectification of electricity by electric bulbs. SHANG-YI CH'EN, *University of Oregon.*—Rectified current derivable from an electric bulb when it is painted with Chinese ink has been observed. The rectifying characters of twenty-three ordinary electric bulbs made by eleven different companies were studied. It was observed that the polarity and the amount of the rectified current derived from the surface of the bulb depends mainly on the gas pressure inside the bulb, the operating voltage, the size and the material of the glass bulb.

(1) The relation between the amount of rectified current (hereafter called output) and the different chemicals painted on the bulb, (2) the output at different locations on the bulb, (3) the relation between output and the temperature of the bulb, (4) the relation between the output current and impressed voltage, (5) the relation between the polarity and the amount of output with the pressure inside the bulb, and (6) the effect of the nature of the coating in the interior surface of the bulb on the polarity of the rectified current, have been studied. An explanation to account for the phenomena has been attempted.

Some effects of intense radiation on ionic crystals. FREDERICK C. BROWN, *Reed College.*—It is known that the pulse-height response of a silver chloride crystal counter can be improved by exposure, while cold and with the collecting field off, to gamma-rays from radium. Results are presented for the exposure of two rather poor counting samples grown from the melt. Upon applying the collecting voltage to a sample after a long gamma-ray exposure, a rapid decrease in pulse height is observed. In terms of the band picture of electrons in a solid, the improvement is usually interpreted as due to the filling of traps—the rapid decrease is believed to be due to the release of trapped electrons by thermal action and the setting-in of polarization. On the basis of a simple analysis of the polarization field set up per unit time and from the initial decrease in pulse height with time, one can estimate the trap depth below the conduction band. In the case of the two samples tested the trapping levels appear to be very shallow, of the order of a few tenths of an electron volt.

A comparison of stars produced by protons and neutrons. LAWRENCE S. GERMAIN, *Reed College.*—Nuclear explosions (stars) initiated by protons and neutrons of known energies have been studied in nuclear emulsions. The emulsions were exposed to high energy protons and neutrons from the Berkeley 184-in. cyclotron. We have used proton energies between 100 Mev and 300 Mev while neutron energies of 90 Mev and 260 Mev were employed. Instrument difficulties were encountered in the measurement of the neutron flux which cast doubt on some of the experiment results; however, the results were quite striking and further experiments are in order to prove or disprove them. They showed the cross section for star production by protons to be considerably greater than by neutrons of the same energy. No explanation has been given for this result and it needs further study.

Recent additions to our knowledge of mesons. BORIS JACOBSON, *University of Washington.*—From the fact that a τ-meson apparently does not decay into two π-mesons, it can be inferred that the τ is either a vector or a pseudoscalar particle. If it is assumed to have a Yukawa-type interaction with nucleons, the coupling constant can be adjusted to give the observed 10^{-9} second lifetime for decay into three π's. A crude calculation yields $(g_\tau)^2 \sim 10^{-5}(g_\pi)^2$. This value is much too small to explain the abundance of τ's if they are produced in nuclear events.

Negative ion formation in gas discharges. MELVIN A. HARRISON, *University of Washington.*—When measuring ionization coefficients over certain ranges of E/p in an electron attaching gas, one must take into account the electron attachment. Likewise when measuring attachment coefficients, one must take into account the production of new electrons by ionization. Previous investigators have evidently not done this. We have measured both of these quantities simultaneously over a limited range of values of

E/p. Work has been carried out in oxygen, a known attaching gas. The results show that the probability of ionization (α) does not fall rapidly at low E/p, but has a value considerably higher than previously reported.[1] Also the results show that the probability of attachment (η) does not decrease rapidly to zero at high E/p values as given previously,[2,3] but remains appreciable to much higher E/p values than formerly supposed.

[1] K. Masch, Arch. Electrotech. **26**, 587 (1932).
[2] N. Bradbury, Phys. Rev. **44**, 883 (1933).
[3] R. H. Healey and J. W. Reed, *The Behaviour of Slow Electrons in Gases* (The Wireless Press, Sydney, Australia, 1941).

Current activities at the University of Washington cyclotron. GEORGE W. FARWELL, *University of Washington.*—Operational tests are being conducted on a 60-inch conventional type cyclotron. Internal ion beams of several hundred microamperes of deuterons or hydrogen molecular ions have been obtained. A magnetic-field shimming program to increase the ion yield at the proposed exit radius (25½ inches) is under way; the largest beam obtained to date at this radius is about 60 microamperes. Conventional cone and hooded ion sources have both been used successfully. The grounded-grid-oscillator circuit provides a very stable rf system. An rf deflector is under construction. Present experiments utilizing the internal beam include studies of excitation functions and inelastic scattering; Cu^{64} has been produced for use in Čerenkov radiation and positronium experiments. Work with 30–40 Mev neutrons is planned. Magnets for beam analysis and reaction product analysis will be located in a low-background region in a building addition now under construction. Nine graduate students in physics are associated with the cyclotron construction and research programs. Technical details of the cyclotron will be described elsewhere.[1] The project has been partially supported by the ONR and the AEC.

[1] F. H. Schmidt *et al.*, "University of Washington 60-In. Cyclotron," Berkeley meetings of the American Physical Society, December, 1951.

Recent experiments with positrons. ROBERT BROCK, *University of Washington.*—Recent experiments related to the annihilation of *ortho-* and *para*-positronium are discussed. M. Deutsch has verified the effect of electron exchange with odd-numbered-electron molecules causing the *ortho-* to *para*-transition with subsequent rapid annihilation. The energy difference between the $^3S_1$ and the $^1S_0$ ground states has been measured by Deutsch by measuring the change in decay rate produced by the (quadratic) Zeeman Effect.

Positronium experiments are being conducted at the University of Washington, and are briefly described.

The afternoon session began at 1:00 P.M. with a business meeting, presided over by DR. WILLIAM L. PARKER, *Reed College*, President of the Section. On motion made by T. R. ELLICKSON, a rising vote of thanks and appreciation were given to the local committee for its fine arrangements and to the authors of the excellent list of papers. The motion was passed unanimously.

Following the business session, visitors were given conducted tours of the University of Washington cyclotron and Physics Laboratories.

FRED W. DECKER, *Secretary*

AMERICAN JOURNAL OF PHYSICS VOLUME 20, NUMBER 5 MAY, 1952

Proceedings of the American Association of Physics Teachers

The Chicago Meeting, October 26–27, 1951

THE meeting of the American Association of Physics Teachers held in Chicago on Friday and Saturday, October 26 and 27, 1951, celebrated the Twentieth Anniversary of the American Institute of Physics. The AAPT is a Member Society of this Institute.

The following program of 11 invited papers and 19 contributed papers, many including demonstrations, was presented. All of the meetings, and particularly the Friday evening session, were well attended. Many of the papers were extended in the question period; in some cases an arbitrary time limit had to be set. Several of these papers will be printed in the *Journal*.

The Saturday morning session was planned to interest the physics teachers in the high schools. Special invitations were sent to about 100 teachers in the Chicago area. The panel on "Cooperation" created interest and has resulted in a second meeting of the two groups which was held at the Museum of Science and Industry on December 15. Specific plans of cooperation have been developed.

C. J. OVERBECK
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Invited Papers and Special Events

A. Photoelasticity and photoplasticity. M. HETENYI, *Northwestern University Technological Institute.*—A demonstration lecture on methods of stress analysis by polarized light.

B. Solar prominences in motion. R. R. MCMATH, *The McMath-Hulbert Observatory.*

C. Ultrasonics. H. K. SCHILLING, *Pennsylvania State College.*

D. Phase difference between ordinary and extraordinary beams. J. G. WINANS, *University of Wisconsin.*—A demonstration.

E. National educational program. (a) The National Science Foundation: Its organization and purposes. ALAN T. WATERMAN, Director, *National Science Foundation.*—The background leading to the enactment by Congress of the National Science Foundation Act of 1950 was discussed and mention made of the principal organizational features, including the National Science Board, the Directors, and the technical divisions. The two major missions of the National Science Foundation are support of basic research in the medical, biological, mathematical, physical, and engineering sciences, and the development of a strong graduate fellowship program. The Foundation's concept of its role in the development of a national policy for the promotion of basic research and education in the sciences was touched upon. Preliminary thought has been given by the staff to ways and means of according recognition to teachers of science for their accomplishments and of providing opportunities for them to keep up-to-date with findings in research, and this was described briefly. Most recent developments relating to the National Science Foundation were reviewed. **(b) The Fulbright program.** WALTER JOHNSON, *University of Chicago.*

F. Some notes on flight at and near the speed of sound. HERBERT A. HOOVER, Aeronautical Research Pilot, *NACA Flight Operations, Langley Field, Virginia.*

G. Panel—Cooperation of high schools and colleges on problems of physics teaching. WILLIAM F. EINBECKER, *Highland Park, Illinois High School,* RALPH LEFLER, *Purdue University,* V. L. BOLLMAN, *Occidental College.*

H. Principles of color photography. ADRIAN TERLOUW, *Eastman Kodak Company.*—A demonstration.

Contributed Papers

1. Some applications of physics in the manufacture of electron tubes. DONALD C. MARTIN, *Marshall College.*—In the design of electron tubes the laws of thermionic emission and the potential distribution between electrodes are important in determining the operating characteristics of the tubes. Also approximate theoretical equations can be used in calculating the necessary changes in tube structure to produce given changes in the common tube parameters of amplification factor, transconductance, and plate resistance. In the manufacturing process a knowledge of the physical properties of materials is important in such matters as the fabrication and assembling of various parts of tubes, and providing proper cathode emission, satisfactory heat transfer, and maintenance of a good vacuum. Also it is important to know what causes such faults as grid emission and secondary emission from anodes and how these factors can be eliminated or reduced. Finally, in the testing of materials and the operating characteristics of tubes, many common physical and electrical measuring instruments are used, as well as some designed for special purposes.

2. Hauksbee's early 18th century triboelectric generator. DUANE ROLLER, *Wabash College,* and DUANE H. D. ROLLER, *Harvard University.*—Some of the factors that favor experimental research leading to important discoveries are well illustrated by Francis Hauksbee's development of the earliest mechanical, electrostatic generator used to investigate the properties of electricity (1705–09). With this generator Hauksbee made observations that would have been difficult or impossible without it. For some of the resulting discoveries he has not received adequate credit; in truth, he missed the significance of some of his observations, but no more so than did certain later investigators who have been given the credit for their discovery. Despite the easy availability of published descriptions, Hauksbee's generator did not come into general use immediately, although it would have facilitated such subsequent work as Gray's pioneer study of electrical conduction (1729–36). Various factual errors and misinterpretations of Hauksbee's work appear in practically all current history books and textbooks that mention him at any length.

3. Technical transfer-training technique. FRANK A. PETRY, *West Los Angeles, California.*—Teaching techniques as exhibited in many classrooms have often not kept pace with the advances of modern industry. Much emphasis has been placed upon theory, but little has been done toward giving a student the type of training which will prove most valuable in later work in industrial research. The importance of shopwork has often been neglected in the physics course; conversely, the importance of physics has often been neglected in the shop course. Thus an educational problem is manifested. The problem is how to effect more efficient cooperation between school shop courses and school physics courses in the teaching of modern technical developments.

4. The contribution of the joint program of the Office of Naval Research and the Atomic Energy Commission to the supply of trained scientific workers. WILLIAM E. WRIGHT, *Office of Naval Research, Washington, D. C.*

5. Some considerations of an employer hiring physicists for applied research. SEVILLE CHAPMAN, *Cornell Aeronautical Laboratory.*

6. Modes, nodes, and antinodes. ROBERT W. YOUNG, *U. S. Navy Electronics Laboratory, San Diego, California.*—Student comprehension of certain facts of acoustics is handicapped by conflicting meanings of words used in the teaching of elementary physics; action to alleviate the handicap is here suggested. In many textbooks the words partial, overtone, and harmonic are treated as synonymous with normal mode of vibration. That normal modes are not always harmonic is illustrated by data for an actual piano string for which the progressive "inharmonicity" is such that the *fifteenth* mode (for example) has a frequency more than sixteen times that of the first mode. Measurements reported here for the playing of a cornet illustrate that its steady sound is not (in the sense of the erstwhile synonyms listed above) a "note compounded of the fundamental and of the partial tones, or overtones, because of the segments."

Vibration antinodes are not necessarily equally spaced in tubular resonators of arbitrary cross section. Textbooks often discuss a just scale based on a middle C of 256 c/s, even though there is practically no experimental evidence that just scales are employed in actual music. Moreover, with the standard A of 440 c/s agreed upon internationally for musical performance, plus equal termperament, middle C has a frequency of 261.6 c/s.

7. The use of cine-loops in physics teaching. FRANCIS E. THROW, *Cornell College.*—The central problem of physics teaching is to impart a clear and vivid understanding of the basic concepts, and in many of these concepts motion is an essential factor. Such cases cry for an animated diagram which runs for as long as students and teacher desire. This need is filled by the cine-loop—a very inexpensive device which is readily homemade.

8. Apparatus for radioactivity measurements in the undergraduate laboratory. RALPH A. LORING, *University of Louisville.*

9. Animated diagrams for physics demonstrations. ROBERT PETRY, *University of the South.*—Preliminary motion pictures of various subjects of interest in the teaching of physics have been made sketchily to test the suitability of these topics for more permanent filming. These include the rotating magnetic field of the induction motor, two simple harmonic motions at right angles, transverse standing waves, longitudinal standing waves, wave fronts in doubly-refracting crystals, and Young's experiment on interference.

10. Intermediate laboratory experiments in the kinetic theory of gases. CARL E. ADAMS and E. G. ANDRESEN, *University of Louisville.*

11. How to solve a physics problem—a suggestion. LEO SEREN, *Florida State University.**—The major part of solving any physics problem consists of a precise understanding and formulation of the problem. As an aid to solving a physics problem, the following questions may be asked: (1) What are the fundamental quantities with which we are dealing? (2) Precisely what is the problem? (3) What are the rules that we must adhere to when working with the known and unknown quantities? (4) What principles of physics can be used in solving this problem? (5) Can we express the unknown quantities in terms of the known quantities by means of the principles just enumerated?

It is the author's contention that these questions can be answered best by relying heavily on logical thought and that it is not the ability to memorize facts and formulas but the ability to think rationally in terms of the principles of physics, which enables a student to solve physics problems. The two-body collision problem was discussed as an example.

* Now at the Naval Research Laboratory, Washington, D. C.

12. Ultrasonics as a subject in the physics curriculum. PAUL J. ERNST and CHARLES W. HOFFMAN *Temple University.*—The great progress of ultrasonics during the last few decades and its numerous applications to science and industry in peace and in war make it desirable that the physics student be acquainted with at least the elements of this modern branch of physics. Nevertheless, in most of the universities and colleges ultrasonics has not been a part of the regular course of study in physics up to now. This paper gives a concise account of what both physics and industry would gain by the introduction of ultrasonics into the physics curriculum.

13. An undergraduate course in radiation physics. CLAYTON M. ZIEMAN, *Wabash College.*—This course is based upon a suggestion made by Caswell and Gordy.[1] Much of it centers around the derivation from fundamental principles of the one-dimensional wave equation, its solution, and the evaluation of the integration constants in terms of the initial and boundary conditions. Elementary vector theory and Fourier analysis are introduced as required. Laboratory and demonstration experiments run simultaneously with the lectures. To provide flexibility, several reference books, rather than a single textbook, are used.

[1] Am. J. Phys. 13, 315 (1945).

14. Bibliography with annotations for science in general education at the college level. VADEN W. MILES, *Wayne University.*—This bibliography of selected published references, limited chiefly to the years 1945–51, inclusive, is a guide for teachers and administrators who are developing programs of science for general education. Annotations of 152 books and articles appear under one of the following headings: Committees, Conferences, Symposia; Status and Trends; Methods of Science; Content and Organization; Evaluation; History of Science; Junior College; Pre-Professional Science; Teacher Education; Pre-College Science; General Readings. Copies may be obtained from the author on request.

15. From simple harmonic to wave motion. C. J. OVERBECK, *Northwestern University.*—The vibrating-spring simple harmonic motion experiment is usually included in the experiments performed in the beginning college physics laboratory classes. It is important also to establish the connection between such "particle" motion and wave motion. Apparatus designed for this purpose was described. The apparatus consists of a 9-kg bob at the end of a long spiral spring. A stylus attached to the bob marks a trace of its motion on paper tape. This tape is drawn horizontally past the stylus at constant speed by a synchronous motor. Frictional losses have been reduced in the design so that a pattern 200 wavelengths long may be traced before the amplitude attains half value. The resulting wave trace provides data to study the progressive wave equation.

16. A wave-motion slide rule. J. D. RICHARDS, *Northwestern University.*—In the study of wave motion at the first-year college physics level, confusion sometimes results over the student's failure to recognize completely the connection between simple harmonic motion and wave motion. Simple harmonic motion may be represented in two di-

mensions by plotting the displacement *versus* the time. However, in the study of sine wave motion produced by a simple harmonic vibrator, the student must examine the simultaneous displacements of many particles in which displacement is a function of both the position of the particle and the time. This is a three-dimensional picture and is usually represented by selecting an arbitrary time and plotting the displacement *versus* the horizontal position of the particle. This plot is similar in appearance to the plot of simple harmonic displacement *versus* time since both are sinusoidal. The unwary student is likely to confuse the two and make the common error of using wavelength and period interchangeably. To resolve this confusion the author has devised a type of pictorial slide rule which allows the student to hold any one of the three variables constant and examine the relationship between the other two.

17. Modification of Hebb's method of measuring the open-air velocity of sound. HARALD C. JENSEN, *Lake Forest College.*

18. Dissecting the cathode-ray oscilloscope. ROSE A. CARNEY and JOHN J. SPOKAS, *St. Procopius College.*—A five-inch cathode-ray tube with its associated power supply and sweep circuit has been designed and built at our college. This apparatus has been specially designed for classroom demonstration with such special features as a completely exposed tube and an extremely slow sweep; the circuit design provides for sufficient electrostatic stability so that an ac wave pattern can be obtained on the exposed tube. Simple experiments designed to demonstrate the properties of the cathode-ray beam were performed on the tube which will be in full view of the audience while descriptions are being made. The sweep circuit was discussed and demonstrated. Finally, the use of the oscilloscope was demonstrated (1) in studying phase relationships between elements of an ac circuit, (2) in calibrating an oscillator using Lissajous figures, (3) in studying very fast chemical reactions taking place at the electrodes in various electrolytic cells, and (4) in studying sound with reference to pitch, loudness, timbre, and beats.

19. Demonstration of the timbre of sound. ROSE A. CARNEY and JOHN J. SPOKAS, *St. Procopius College.*—Using a seven-inch cathode-ray oscilloscope in conjunction with two variable frequency audio oscillators a very instructive classroom demonstration can be made of the property of sound known as timbre. In this demonstration a pure note emitted by one of the oscillators was heard through a loud-speaker at the same time it was observed on the screen of the oscilloscope. Then the second harmonic of this note was emitted by the other oscillator and was seen and heard simultaneously as before. Then these two notes, the fundamental and its second harmonic, were sounded together and the change in the oscilloscope pattern corresponding to the change in the quality of the tone was clearly noted. This was repeated for a fundamental and several other harmonics. Then a note having timbre was sounded so that one could see that its wave form was complicated as in the case of a fundamental sounded simultaneously with its harmonics. For example, a note played on a violin was observed as well as the same note sung by the human voice. After such a vivid demonstration one better understands the textbook statement, "The quality or timbre of a sound is determined by the number of harmonics present and their respective intensities."

It seems almost ludicrous that precisely in the same years or decades which let us succeed in tracing single, individual atoms and particles, and that in various ways, we have yet been compelled to dismiss the idea that such a particle is an individual entity which in principle retains its "sameness" for ever. Quite the contrary, we are now obliged to assert that the ultimate constituents of matter have no "sameness" at all. When you observe a particle of a certain type, say an electron, now and here, this is to be regarded in principle as an isolated event. Even if you do observe a similar particle a very short time later at a spot very near to the first, and even if you have every reason to assume a causal connection between the first and the second observation, there is no true, unambiguous meaning in the assertion that it is the same particle you have observed in the two cases. The circumstances may be such that they render it highly convenient and desirable to express oneself so, but it is only an abbreviation of speech; for there are other cases where the "sameness" becomes entirely meaningless; and there is no sharp boundary, no clear-cut distinction between them, there is a gradual transition over intermediate cases.—ERWIN SCHRÖDINGER, Science and Humanism (1951).